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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

Mirabeau B. Lamar

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THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XIX, Number 1

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This bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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STUDENT REACTIONS TO ADMINISTRATIVE TECHNIQUE IN CERTAIN MATHEMATICS COURSES

BY GORDON H. GRAVES
Purdue University

Whether we like it or not, students do a lot of talking about us. Our characters as well as our mannerisms are under daily observation and discussion. Reactions to the way we administer our courses are freely interchanged, but, of course, rarely with us unless we make the occasion for it. It will occur to anyone who stops to think about it that these young people have a vital interest in the educational processes to which they submit. Though they are not always conscious of the importance of this interest, they are continually making keen observations on relevant matters, characterized by much better judgment than we sometimes give them credit for. It would seem that here is a valuable source of criticism which it would be well to consider.

This was called to my attention in a talk by J. M. Artman ten years ago, and ever since I have asked my students for criticisms at the end of the semester. These papers do not need to be signed and are kept by one of the students until the semester reports are out of my hands. They have nearly always given an impression of frankness and have been valuable in interpreting viewpoints and in suggesting procedures. Some of these criticisms are talked over with members of the succeeding class who thus have their attention called immediately to the desirability of observing and evaluating what is going on. Also the way is opened for further discussion with individuals and even with the class at available times.

Some fairly stable cleavages in student reactions are apparent after these years, and of them it may be of interest to speak.

The most striking difference is between students who want to be self-directing and those who want the teacher to assume responsibility for their activity. At the beginning of the semester, I tell classes that it is only rarely that I shall expect the problems they do at home to be handed in; that it is very necessary that they solve problems and that those they have trouble with should be brought to class or conference but that the responsibility for this is theirs. This is usually received with approval at the time and many still approve it at the end of the semester, but there is nearly always a group that argue: "We are only a little beyond high school and can not be expected to stick to our job faithfully enough to learn what we might. It would be better to have to hand in problems every day even though you merely threw them in the waste basket, for we would never know when you might look at them and it would keep us working."

There is always disagreement on the grading system. I try to have each course divided into "jobs," which may be no more than the chapters in the text, and every job must be done in order to receive credit in the course. The division into jobs is approved by the students without exception and there is even no objection to the requirement that every job must be done. But I assign no grades for these jobs, merely telling each student at the end of the time devoted by the class to the job whether he is "clear" or not. This is unusual for them and requires explaining. I point out that working for a grade is a distraction from the main task, that I am asking them to do their personal best and that I am not trying to whip them all up to a standard which only a few can attain nor hold these down to an accomplishment less than what they ought to do. By the end of the semester a substantial majority is in favor of the plan, but there are nearly always some who "like to know where they stand." It does not satisfy them to be told that they have more definite information when they know just which of the jobs they have finished to date than if they had a letter or numerical grade. It is often evident that these students want to temper their effort to what they discover the teacher's standard to be or wish to compare themselves with other students in the class. Some remark that the plan is good, but that it places a course where it is used at a disadvantage if in other courses class exercises are graded so that means are at least provided for computing an average. The idea is, apparently, that courses compete among themselves for the student's time and attention. Students who are not clear in a job are invited to make appointments to go over their difficulties and may take a test over the job within a month. This provides them with a second opportunity to clear their record. If it is not done then there is still a chance at the time of the final examination. Students appreciate these conferences and often suggest that they be made compulsory. The make-up examination has had only one expression of disapproval so far as I recall. This was that it is not fair to the student who does not get behind. The standpoint seems to be that of the older brother in the Parable of the Prodigal Son.

Assignments come in for criticism. Every semester some students suggest that "it would make it so much easier" if, in making an assignment, the teacher would go over it with them before asking them to study it. We often discuss this matter and it appears to be a carryover from a common high school experience. They usually see that much time can be wasted in attempting to go over material before they have themselves grappled with the problems in it and located the questions they wish to discuss. I agree with them that just to toss out "the next three pages" is unfair and, of course, usually do indicate the bearing of the next assignment on the work they have been doing or its place in the general plan of the course.

It is common for students to object to "theory." In a technical school many apply a severely "practical" test to every item of their intellectual fare. But it turns out on inquiry that "theoretical" has a very variable connotation. Some mean that they do not like to prove theorems. They are impatient of any efforts to get them to understand principles and relations when after all "there is a formula somewhere, isn't there, in which one merely need substitute?" Some have an exaggerated idea of the ability of mathematics to provide them short cuts to results and some expect to see just how each matter we take up will enable them to demand more pay. I have found some who meant by "theoretical" general statements as opposed to concrete instances, for whom, for example, 7 miles + 3 miles would be acceptable but $7a + 3a$ anathema. In this connection it must be borne in mind that many mathematics courses suffer from incomplete motivation. So often we are trying to provide beforehand for needs in courses to come later that it is quite natural for students to want details.

There is considerable variation in the reaction to classroom procedure. Many express themselves in approval of class discussion because of the variety of viewpoints it reveals. Some think it a fine thing to draw out a slow student with patient questioning and others regard it a waste of time. Some are very severe and urge that if a student can not answer questions he should be let severely alone as a lesson that he ought to study harder. I am continually surprised at the number of those who ask for more board work. Apparently many have become adjusted to a routine of spending part of each class period at the board "doing problems." Of course, it is valuable for a student to present a discussion from the board, particularly if he explains as he writes, but it takes so much time to get the experience necessary to do this well that it is available to only a limited extent in all but the smallest classes.

For several years I have had a group of prospective teachers of mathematics in a course in synthetic geometry. Here it has been my object to assist them in gaining some background, so there has been an especial effort to notice pertinent historical and philosophical material. Most of the students consider this both interesting and valuable, but some regard it as an intrusion and one even said that it served as a means of avoiding giving answers to "practical" questions.

Some will ask what values inhere in this collecting of criticisms when there is obviously bound to be considerable conflict of opinion and when only few suggestions at best can be followed. The answer is to be found in the improvement in the attitude of both teacher and student toward their coöperative effort. If the student is encouraged to express himself, even though anonymously, there is less "griping" and his attention is called to the fact that it is his class after all and not something put over on him. Further, it is extremely valuable for the teacher to know the student reaction to his procedure. Unpopular techniques may be faulty and these criticisms afford a means of knowing where changes are most likely needed and of explaining things that are misunderstood. The consensus of opinion frequently serves as a vote of confidence and there is every now and then a suggestion of great value either as a basis for experimentation or as a correction of some detail which was annoying but of which the teacher was quite unaware.

ATTEMPTED REVISION IN HIGH SCHOOL ALGEBRA*

BY MRS. HARRY BREWTON
Hemphill, Texas

Careful study during the past fifteen years of our American schools and comparison with European schools reveal the vital need of the reorganization of mathematical instruction.[1] It is true that for the past thirty years there has been no lack of interest in the subject of the teaching of algebra. Articles, books, monographs, and reports on this subject have flooded the country. The more recent demand for improvement of the teaching of algebra has been influenced by the Perry movement in England; the address of Professor E. H. Moore before the American Committee on Mathematical Requirements; Dr. Klein's recommendations; the report of the International Commission on the Teaching of Mathematics; the report of the Committee on the Reorganization of Secondary Education; the Junior High School movement; and the nationwide discussion of curriculum revision.

According to the Seventh Yearbook of the National Council of Teachers of Mathematics, "The forces that have given rise to the present tendencies in the teaching of algebra are represented in the persons of school administrators, educational sociologists, and philosophers of education." Permit me to add to this the laymen of our communities. The administrators ask: "What pupils can study algebra?" The educational sociologists ask: "Of those who can study algebra, what pupils should be permitted to take it?" Philosophers of education ask: "What basic method should

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*Read before the Texas State Teachers Association, Galveston, November 30, 1934.

[1]The Reorganization of Mathematics in Secondary Education, Report of National Committee.

underlie the teaching of algebra?"[2] The laymen ask: "Why do our children study algebra at all?"

Until recently mathematics has been preserved in the school curriculum like the ancient Greek and Roman classics. The courses given were made up of many antediluvian problems which were of no value at all. Only recently a deputy state superintendent visited a school in Texas where the teacher was stressing the hare and hound problems which high school students were required to work twenty years ago. Puzzle problems have no place in the classroom but should be used to furnish amusement in the mathematics club work. Because mathematics was a highly respected and protected subject, we felt that its place in the high school curriculum was assured; but a reaction came, and today we as teachers of mathematics must justify the teaching of our subject.

There are quite a few people today who are unfriendly to mathematics, and they are constantly discouraging the idea of requiring any mathematics beyond the seventh or eighth grade. There has been quite a bit of criticism of the fact that mathematics does not "carry over" into life. The demands made upon modern mathematics by our present civilization, which includes many sciences involving a knowledge of mathematics, have made necessary a change in our textbooks, in our methods of teaching, and in our curriculum. The hardest criticism for mathematics teachers to face is that there are too many failures in algebra. We know that this criticism is just, and it is our business to find a remedy for it. The causes given for the larger number of failures are: (1) poor teaching; (2) the content of the algebra course is too remote and too unpromising to enlist the interest of even the more intelligent pupils; (3) the school population of today is far different from what it was twenty years ago.

There is no doubt that algebra has been poorly taught. In checking over the public school directories of Texas for

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[2]Seventh Yearbook of the National Council of Teachers of Mathematics.

the past several years, I found that less than 50 per cent of the high school teachers of mathematics have any special preparation in their field of teaching. Some teachers choose mathematics because the papers are easy to grade; classes in the higher grades are small; daily preparation requires less time. In many cases it is the fault of the school administrator in that he gives the teachers of the other high school subjects a class in mathematics in order that they may have a full teaching load. The principal or coach has an algebra class because he has little time for the correction of papers and for daily preparation. You as teachers are familiar with many instances of teachers working at night in order that they may stay ahead of their classes, because they live in dread that they will be called upon to solve problems which they do not know how to work. This unpreparedness, together with a mediocre grasp of the subject, has led to quite a bit of teaching by rote and is partly responsible for the poor results in our mathematics teaching.

Our State Department and State Board of Education have taken a forward step in requiring that high school teachers must teach in their fields of preparation and that all teachers must attend summer school in order to keep up with the more modern ideas of teaching. Since most of our teachers are willing to work and eager to learn (as shown by the increased attendance at summer schools), we feel that this movement will lead to better teaching of the high school subjects.

The second contention, that the contents of our algebra courses are too remote and unpromising to enlist the interest of the pupils, will not be entirely removed until we rise to the occasion and vitalize and humanize our work with the algebra classes. The present educational program is child-centered and calls for the adaptation of the subject to the pupil in order that he be better fitted to compete in the present complex civilization. In a proper reorganization to meet the needs of the pupils, it must be agreed as to what the purpose of the various parts of the course is, as to the content best suited to the purpose, and as to the most efficient and economical method of presenting this material.

Before a teacher can make much change in the content of the course in algebra, he must be freed from the requirements laid down for college entrance and for affiliation, from supervisors who block out each week the amount of work that must be covered, and from administrative officers who demand that certain tests must be given at fixed periods during the course. Of course we must prove ourselves capable of handling the situation before we can expect to be permitted to work out courses which we think are best suited to our schools.

The experiment on which we are at work in our school is to so change the content of our algebra course that the pupils will find the work practical and useful. In order that those pupils who expect to attend college and who desire the traditional course in algebra may not lose by the experiment, we are giving a regular course in eighth grade algebra in addition to the new work. We have in the experimental class only those children who have come in from the rural schools and those who will probably not attend college.

We are trying to give to this special group a course which we might call community mathematics or humanized algebra. After all, the school educates only to the extent that it gives understanding of the community and insures more intelligent participation in the activities in which the children will engage. Until recently we were preparing pupils for college; today we are developing them for citizenship. You may ask if we are going to the men on the streets and inquiring of them what problems they use in everyday life, and narrowing our courses to include only these. That is not our objective at all. We are trying to show the children that they do use mathematics in everyday life, to stimulate the child's interest, and to cultivate an appreciation of the fact that the men of the streets could use algebra if they only knew how.

Most of us face the same problem each fall of meeting in our eighth grade algebra classes pupils who have a frightened look in their eyes because they have been told that algebra is so hard that they fear they will not be able to pass the course. We conceived the idea of making the first six weeks a real surprise for them. We collected problems from the homes, the business houses, and the mill. These problems included the housewife's problems of proportions, the farmer's problems of net earnings from his various crops, and problems from the lumber mill, which is the chief industry of our town. When we were figuring bills for lumber for buildings, we used and developed formulas of areas without the pupils being aware that we were really studying algebra.

We then introduced the problems of taxation in our city and county. We worked out a project by which the pupils visit the city tax collector and the county officers to find just how much tax money had been paid in, from what sources it had been derived, and how it had been spent. It was a pleasant surprise to find on the next day that one of the least interested boys in the class had brought the latest edition of the *Texas Almanac,* which gave the State's revenues, the sources from which they were derived, and how they were spent.

As this was the time to teach circle graphs, we introduced them, and the children were really eager to put their information in the form of a graph. While we were making these graphs we were studying the sources from which our county derives its taxes. The children brought to class their parents' tax receipts. We learned how the tax money was spent, the purpose and necessity of taxes, and probable tax changes. Some of the children who had dreaded algebra were delighted and would say that their lessons were just like civics; so you see, even if we had not actually covered the work as laid out by the State Department of Education, we had removed the dread of the course and had caused the children to realize that mathematics is a part of their everyday life.

We spent some time on the study of insurance, which is a vitally important problem to the children whose parents are employed at the mill. Emphasis was placed upon the economic aspects of preparation for home making, budget making, and the keeping of accounts. It would not be unwise to teach a child how to manage his personal finances. Since there is no large trading center in our county, many people purchase from mail-order houses. The children brought their catalogues to class, worked out orders (incidentally getting practice in letter writing), figured the weight of the order, looked up the parcel post zones, and found the amount of postage due on their orders. This seems like a small problem; yet it deals with a problem of modern life. It also introduced an activity on the part of the pupil—a thing for which modern education is striving.

We are also attempting to correlate the work in algebra with the other school subjects by having the pupils bring in problems from other classes. The general science classes are finding considerable need for equations; the home economics classes furnish problems in proportion; the social science classes have been paving the way for the introduction of positive and negative numbers by determining how many years have intervened between events happening before and since the birth of Christ.

In the study of formulas the pupils have learned to derive and apply them. The children already knew how to find the area of a rectangle; from this they derived the formula for the area of a triangle. They used a sheet of paper which they ruled in inch squares, drew a diagonal, and cut the paper along this line. They found that they had two equal triangles which together made the rectangle. They counted the number of square inches in each triangle and found half as many as in the rectangle. They then wrote out their own formulas and were pleased that they had made a discovery. Here was introduced the idea of functional relationships by showing how the area of the triangle changed as the base changed when the altitude remained fixed, and vice versa.

Since the value and meaning of π had always given considerable trouble to most pupils, the children were permitted to discover this relationship for themselves. They brought to class tape measures, cardboard, and compasses. They constructed circles, measured the circumference and diameter, and divided the former by the latter. They found the relationship between the two to be 3.1416; and now there is a definite idea in their minds as to what π really means.

We are trying to teach new things as the need arises. A short time ago we had a problem involving the four fundamental operations. It has always been rather difficult for the pupils to learn to evaluate algebraic expressions in which multiplication, division, addition, and subtraction occur. To make this simple we used red crayon for $+$ and $-$ signs and green crayon for $\times$ and $\div$ signs. We compared these to the traffic lights, with which all high school children are familiar; and in a very few minutes each pupil was able to apply this principle, on which I have known teachers to spend several days.

No doubt we are using poor psychology in trying to undo the methods the students learned in arithmetic by teaching almost the opposite in algebra. They learned to add, subtract, and multiply from the right side in working with numerals. Of course, there is a very fundamental reason why we work from the right in arithmetic, but we should not confuse the child by changing his fixed habits when he begins the study of algebra. We never realized this until a few years ago, when a student, after having studied multiplication in algebra, began also to multiply from the left when he was working with numerals. Since there is no basic reason for changing the habits learned in arithmetic, we should make as few changes as possible in introducing algebra, which is really generalized arithmetic.

No doubt you feel that the type of work we are attempting is very slow; but we trust that it will make it possible for a class of average ability to accomplish more in the course of a year, to cover more topics with greater thoroughness than could be done under the old plan of the teacher's showing the pupils how to work several typical examples, then having the pupils work others by imitation.

The spirit of every mathematics classroom should be one of adventure and exploration, and we should give the children opportunities of assuming a more creative role. After all, the amount of algebra covered is of minor importance as compared with the need of making the pupil conscious of the quantitative aspects of the world in which they live.

"The future of mathematics in the secondary school curriculum depends on the ability of the teachers of mathematics to realize the full value of the subject and its close contact with the structure of civilized progress, and in their ability to so organize and present its content that the student may thoroughly appreciate its historical significance, present potentiality, and future challenge."[3]

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[3]The Mathematics Teacher, October, 1934.

MATHEMATICS CLUBS*

BY WILLIAM C. LARIMER
Denton, Texas

The more progressive educators have promulgated within the last quarter of a century a new field of education augmenting the conventional régime of education. This new field takes into consideration the paramount importance of the social and vocational development of the child. This new field consists of extra-curricular activities. Mr. E. H. Wilds, in his *Extra-Curricular Activities*, chapter one, defines extra-curricular activities as those activities that are outside the regular curriculum, that have sprung up and have developed through the student's own desires and efforts, that are carried on apart from the hours of the regular school program, and that are participated in without the rewards of regular school credit. Now, more than ever before, these educators are coming to realize how out of date are the disciplinary and classical aims of education. A new era has dawned in which the old objectives of education must be discarded and new objectives must be set up.

Before the advent of the twentieth century, there was little need for a program of extra-curricular activities; pupils lived a sufficiently active life to preserve their health, and the social demands made of them were meager. In this new age we find a vastly different situation, for life in the teeming cities of today is far more complex than the life found in the homes of our predecessors. The stupendous industrial activities, the hustle and bustle of our great cities, the concentration of the population within a comparatively small area, the efficiency of the teeming business world, the accumulation of immense wealth, and the ever-increasing amount of leisure time—all demand a new type of training.

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*This is Chapter II of Mr. Larimer's M.A. thesis, entitled "The Stimulation of Interest in Secondary School Mathematics," a copy of which is deposited in the Library of The University of Texas.—Editors.

The administrators of our secondary schools are beginning to realize the great importance of an activities program, but some of them, who have not inaugurated programs of this kind in their schools, are uncertain as to what to include in an activity program. Any person who has made a study of extra-curricular activities can mention a great many activities, but we shall confine ourselves to club work. We shall consider briefly certain fundamental principles of clubs before we make a specific application to mathematics clubs. There are certain principles that must be taken into consideration before any club can function efficiently.

Many of the more conservative educators have denounced the club program in the secondary school because they believe that the students are too immature to discharge the duties inherent in successful participation in club work. Advocates of the new liberalism have cited certain characteristics of adolescent children which to them justify the organization of clubs.[1] During the stage of adolesence, new instincts come into play; new interests develop; and new habits are formed; hence the adolescent child is an impulsive creature, with many interests, strong emotions, and varying moods. Ever seeking an opportunity of expressing himself, the adolescent child likes to undertake things for himself. It is during this highly complex stage of life that the gang spirit will manifest itself by an innate tendency upon the part of the individual to initiate some form of organization, whether directed by himself or by a wise teacher. He is by nature a sociable being; and a thread of egotism invariably manifests itself in his love of approbation, particularly the approbation of his fellows. This love of approbation, one of his controlling instincts, tends to be greatly in evidence when an adolescent child has some responsibility. Some little responsibility will invariably flatter his pride and will enhance his self-reliance. Thus,

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[1]Roemer, Joseph, and Allen, Charles F.: *Extra-Curricular Activities in Junior and Senior High Schools*, p. 104.

we see that the adolescent child is psychologically in a receptive mood for a club movement.

The task of organizing a school club is a very delicate one, for even though the characteristics of the children during this period seem to guarantee the success of the club, there is one rather distressing feature of the children of this period; namely, the unstable nature of their moods.

Let us discuss briefly some of the tests of a school club.[2] The first and most important requisite for the success of any club is that there exist a common interest. This not only applies to the students, but it applies to the faculty as well. The first step is to secure a favorable attitude on the part of the faculty. The faculty under the leadership of the principal should attempt to effect a plan of inaugurating clubs. The most nearly Utopian way of initiating a club is to let the organization evolve from the requests of the students or from the suggestion of a teacher who volunteers sponsorship.[3] The objectives of the club should be based upon the real live interests of those pupils who are requesting the organization of the club. What are the sources of these common interests? The interests may grow out of the curriculum. Certain students may become intensely interested in some subject and wish to delve more deeply into its various phases than the class periods will permit. Here we see the exploratory and experimental aspects of interest manifesting themselves.[4] A club will furnish an opportunity for the children to learn how to work together. It will also allow the children to interpret various phases of a certain field better, for their actual contact with these phases in the club work will make a more lasting impression because the manipulative instinct is allowed to function. Their work is made more vital to them when they are allowed to do their own exploring.

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[2]Fretwell, Elbert K.: *Extra-Curricular Activities in Secondary Schools*, p. 291.

[3]Foster, Charles R.: *Extra-Curricular Activities in the High School*, p. 23.

[4]McKnown, Harry C.: *Extra-Curricular Activities*, p. 93.

Elbert K. Fretwell points out in his *Extra-Curricular Activities in Secondary Schools,* page 291, that the size of the club is of vital importance. The membership should be large enough to provide for group stimulus. Yet it should be small enough to make it necessary for the members to participate as individuals or as members of small groups within a larger group. These questions naturally arise: Upon what bases shall the membership of the club be determined? How shall the size of the club be regulated? There are no set rules governing admission to a club, for these requirements must be determined by the purpose of the club and by the nature of its work. Some clubs require only an interest in the field which the club is designed to cover. Other clubs have more stringent requirements, such as a required number of courses in the field, a certain degree of scholarship, a certain degree of skill, a certain fee, or a certain classification. There is one rule that should be rigidly adhered to; that is, that under no circumstances should the members be admitted to a club by a vote of the students. The reason for this requirement is obvious. Possibly the most liberal entrance requirement is to have all the clubs open to all children possessing the necessary interest and skill. The question of the status of the member who has failed in some of his courses frequently arises in clubs requiring a certain degree of scholarship for admission. One answer tersely states that if the student cannot carry his school work, he should not be allowed to be a member of a club. Some clubs give a student a month's notice, during which he must exhibit some improvement in his work, or he will be summarily dropped from the rolls.

Naturally one would expect that the members should exhibit spirit in the work of the club. This may be brought about by a program which takes into consideration the child's span of attention and interest. There should be an opportunity afforded to do something that will tax his efforts, not unduly, but just sufficiently to permit him to feel a development in power.

Another test of a club concerns its relation to the school. No club should be formed hastily, nor should it exist without a charter from some central organization in the school. It should arise through student interests and should, whenever possible, be linked up with some school subject. The schools, with the pupil's point of view in mind, should develop a club program by arranging a favorable opportunity for their satisfying existence and guidance. If the club activities are worth carrying on in school, they are worth carrying on during school time. Including the club activities in the regular schedule makes them more convenient for the students and for the teachers and encourages a good attitude toward the clubs by adding a certain atmosphere of importance and dignity to them.

Probably the factor most responsible for the success or failure of the club is the sponsorship. Actually the sponsor may be said to "make" or "break" the club. Volunteer sponsors probably make the best ones. The principal may appoint sponsors for the various clubs. This plan is perhaps better than the plan of allowing the members to select their sponsor, for the members will invariably select a very popular member of the staff, and popular teachers are not always good sponsors. Teachers should realize that today, more than ever before, their sphere of influence is not confined within the four walls of their classrooms. The sponsor should be interested, enthusiastic, vivacious, reasonably popular, sympathetic, kind, and unobtrusive. The sponsor should be the guide and counselor for the club members. He should be ready to handle any situation that may arise and know what to advise, when to advise, how to advise, and in what amount to advise.[5]

The policy of having the smallest number of executives and the fewest committees is particularly sound; the committees should be appointed as they are needed and discharged after their tasks have been performed.[6] Many

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[5]Fretwell, *op. cit.*, p. 292.

[6]McKnown, *op. cit.*, p. 96.

clubs have various committees that remain inactive practically the entire year; such a policy is deadening to interest.

Since we have outlined the fundamental principles underlying the organization and the work of clubs in general, let us trace the history of mathematics clubs and analyze the possibilities to be found in them. Probably the first mathematics club to be organized in a secondary school in this country was the one organized by C. W. Newhall in the Shattuck School, a private school for boys in Fairbault, Minnesota, in 1903. In 1913 Miss Marie Gugle organized a Euclidean Club among the boys in Scott High School in Toledo, Ohio. Membership in this club was limited to boys of the tenth to the twelfth grades. In 1914 groups of interested pupils in the Hyde Park and the Bowen High Schools of Chicago began to devote time after the regular school hours to informal discussions and to the investigation of problems that had been suggested during the classroom activities. In 1916 the first mathematics club in Columbus, Ohio, was organized in the Roosevelt Junior High School.

The reader is no doubt surprised to learn of the comparative recency of the founding of mathematics clubs in secondary schools in this country. A brief survey of the extra-curricular activities of secondary schools would reveal a surprising paucity of mathematics clubs. There are several reasons that have been advanced to explain this situation.[7] Miss Reed contends that this paucity is partially due to the teacher's failure to realize the need or the possibilities of such an organization. Many school teachers have found it easier to do things the same old way year in and year out than to be progressive. Some teachers are averse to assuming the responsibility that comes with sponsoring such an undertaking. The teacher-training institutions are coming to realize that today the sphere of the teacher's influence has broadened. These institutions have included courses in extra-curricular activities in their curriculum.

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[7]Reed, Zulu: "High School Mathematics Clubs," *The Mathematics Teacher*, Vol. XVIII (Oct., 1925), p. 342.

Another obvious reason for the rather small number of mathematics clubs is to be found in the scarcity of library equipment for the mathematics department. None of these barriers is impenetrable, for with the more careful training of teachers and with the comparatively low cost of library equipment, these barriers should ultimately vanish.

One occasionally wonders just how a mathematics club is started and how it is organized. Many teachers have doubtless had some pupils linger after school hours to investigate various problems that have been suggested to them. A club can evolve from this highly desirable manifestation of interest. A club will offer an opportunity to bring these enthusiastic pupils from the different classes together at a regular time and direct the interest already created into profitable channels. When interest in mathematics has been aroused to the extent that pupils will remain after school to delve more deeply into mathematics the time is "ripe" for a "sales talk" for the mathematics club. The matter may be taken up by the teacher who is fortunate enough to be detained in the manner described above. He may casually suggest the matter to these interested pupils who linger after school. He may make an informal talk to his classes, or the matter may be brought to the attention of the students during a student assembly. Possibly all three procedures occurring in the order in which they are listed would be very effective.

What may we hope to accomplish by organizing a school club? What shall we set up as our aims? As our purposes? Let us consider briefly the purposes and the values of mathematics clubs as proposed by enthusiastic teachers. Of course, the whole matter may be summed up by simply stating that the purpose of a mathematics club is to stimulate interest in mathematics. But let us be more explicit. The members of the mathematics club, brought together by kindred interest, may explore and discover for themselves the many interesting and worth while pleasures that are to be found in mathematics; they may learn of the struggles that science and mathematics have passed through as they have developed through the ages; they may awaken to the utter indispensability of mathematics to the existence of our modern civilization; and they may penetrate more deeply into the field of mathematics, and come to revere its infinite vastness, to realize our debt to the great mathematicians who represent human beings like themselves, to appreciate the absolute truth and the supreme beauty of mathematics and to marvel at the austere grandeur of the symphony of the universe.

Many teachers point out that club work will tend to establish a habit of reading mathematical articles in magazines, and that the recreative material found in the field of mathematics will serve for a worthy use of leisure time, one of the cardinal principles of education. Some teachers see in certain pupils the teachers of tomorrow. It is a purpose of the mathematics club to inspire these future teachers with the nobler phases of the subject so that they in turn can inspire their pupils.

Many skeptical people will believe that the values and purposes listed above are merely the pardonable exaggerations of over-enthusiastic teachers. In order to allay this feeling, let us note what some students have said in commendation of the clubs and their work:[8]

> The mathematics club has helped me in three distinct ways: First, it has given me a knowledge of the less common geometric and algebraic problems and the easier methods of working out those problems. Second, it has brought me into contact with other fellows interested in scientific and mathematical work; and, third, it has enabled me to know of new inventions by hearing pupils give talks about them; therefore, I am better able to understand lectures given on science and mathematics and to comprehend articles written on these subjects.
>
> I have been more interested in geometry than ever before.
>
> The mathematics society has given me something to study for.
>
> I enjoy the freak geometry theorems.
>
> I enjoy what I have done. . . . It has helped me to review my work and has given me a real desire to teach geometry.

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[8]Snell, C. A.: "Mathematics Clubs in the High School," *The Mathematics Teacher*, Vol. VIII (Dec., 1915), p. 75.

Since the fundamental principles of organizing a club do not vary very much, it is necessary only to outline the salient features of a mathematics club. We have already considered the initial steps, the capitalizing of certain interests.

There is a wide range of names. The name may be merely the *Name of School* High School Mathematics Club. The following variations will deserve consideration. The name may be derived from Greek letters; it may be derived from the names of mathematicians, such as Pythagorean, Euclidean, or Archimedean Club; or the name may be derived from mathematical figures, such as the Magic Circle, the Triangle, the Octagon, or the Hypocycloid Club.

Meetings should be held often enough to prevent a diminution of interest because of inactivity, yet they should not become too frequent. Possibly once every two weeks would be about the correct frequency of meetings. The time of the meeting should be, if at all possible, during the regular school hours; by all means in the activities period or the hour set aside for club work, if there is such a provision in the school program.

The basis of membership will have to be determined by the situations confronting each individual club. Practically any of the suggested entrance requirements mentioned in the first part of this chapter may be adopted.

The officers should consist of a president, a vice-president, and a secretary-treasurer. It is not necessary that the officers be known as the president, vice-president, and the secretary-treasurer. They may be known by certain mathematical terms or figures.[9]

The number of committees may be determined by the different circumstances. Since we should reduce the number of committees to a minimum, perhaps it will be sufficient to have a program committee and a social committee, the latter committee being appointed shortly before a social event of the mathematics club. Some clubs have been known

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[9]McKnown, *op. cit.*, p. 96.

to have as many as eight committees. These committees are committees on entertainment, library collections, history, vocations, problems, student help, contests, and prizes. The work of the program committee should consist of having conferences with the teachers reative to the subject matter of the programs and of organizing this material and distributing the burden of the program in the best manner possible.

To the casual observer who has no particular interest in mathematics, the matter of mapping out a program presents an almost insuperable difficulty. They ask how may we find enough material from the field of mathematics to make a program. There are several principal sources of program material: talks, field trips, projects, demonstrations and experiments, recreations, contests, games, and plays.

Participation in the work of the club is necessary to sustain interest. Papers open up an avenue to active participation. Topics that may be considered may be classified under the following heads: bibliography, history, advanced mathematics (for the high school student), applications, instruments, and recreations. A partial list of great mathematicians would include Euclid, Newton, Vieta, Stevin, Oresme, and Recorde. Mathematics may be called the inheritance of mankind, since it is probably the oldest science known. Starting with the instinctive sense of number in man and beast, mathematics has developed by slow degrees until now it is the most powerful of the sciences. The following are a few of the topics on the history of mathematics that may be considered: Hindu-Arabic numerals, number systems, geometry in ancient Egypt, and zero and the principle of local value. In the field of advanced mathematics there is a vast number of possible topics. We need only to go to the algebras and trigonometry books of the first year of college mathematics to get materials for topics. For topics on the applications of mathematics in the various fields of endeavor, the club may either have its members present topics or it may invite someone to talk at their meetings. The members of the faculty of the mathematics department, the science department, the mechanical drawing department, the commerce department, or the arts department are available speakers who could outline the use of mathematics in their fields, while speakers who are not directly associated with the school may be available. In this last group belong the local engineers, the banker, the business man, the photographer, and even the lawyer. Over two hundred topics for mathematics club programs may be found in the following brief bibliography:

Hansen, Lena B.: "Creating Interest through Special Topics," *The Mathematics Teacher*, Vol. XXIII (Jan., 1930), pp. 3–6.

Hoag, Ruth: "Program Material and Some Types of Programs Which Might Be Undertaken by High School Mathematics Clubs," *The Mathematics Teacher*, Vol. XXIV (Dec., 1931), pp. 492–502.

Newhall, Charles W.: "A Secondary School Mathematics Club," *School Science and Mathematics*, Vol. XI (June, 1911), pp. 500–509.

Read, Cecil B.: "Mathematical Fallacies," *School Science and Mathematics*, Vol. XXXIII (June, 1933), p. 585.

Shoesmith, Beulah I.: "Mathematics Clubs in Secondary Schools," *School Science and Mathematics*, Vol. XVI (Feb., 1916), pp. 106–113.

Steward, Margaret: "The Mathematics Club of the Pontiac High School," *The Mathematics Teacher*, Vol. XXIII (Jan., 1930), pp. 25–29.

Taylor, Helen: "The Mathematics Library and Programs," *School Science and Mathematics*, Vol. XXX (June, 1930), pp. 626–634.

Vallandingham, J. T.: "Use of Papers Before a Mathematics Club to Contribute to the Variety of the Program," *School Science and Mathematics*, Vol. XXI (Dec., 1921), pp. 817–823.

Wheeler, Albert Harry: "Mathematics Club Programs," *The Mathematics Teacher*, Vol. XVI (Nov., 1923), pp. 385–393.

Some members of the club or of the mathematics staff could demonstrate certain mathematical instruments and explain their use. Instructions in the use of the following instruments could be given: abacus, slide rule, transit, adding machine, and the calculating machine.

Projects have been taken up by mathematics clubs and have been carried to a successful conclusion. Some clubs have (1) compiled a dictionary of terms and symbols used in high school mathematics; (2) made a mathematical scrap book either upon an individual basis or upon a group basis to be presented to the school library; (3) constructed mathematics instruments to be used later in field work; (4) written a short history of mathematics to be presented to the school library; (5) constructed sets of models to be used in the plane geometry and solid geometry classes; and (6) edited a mathematics paper or a mathematics section in the school paper.[10]

The demonstrations and experiments that may be performed as a part of the club program include the demonstration of the sphere, the ellipsoid, the paraboloid, and the hyperboloid by means of soap bubbles; the finding of the value of pi by chance; the demonstration of the operation of the slide rule; the use of the steel square; and the folding of paper in plane geometry.[11]

Mathematics teachers would like to know where a mathematics club can go on a field trip. The club could go on a surveying trip and measure the height of objects which have either accessible or inaccessible bases. Among the objects that could be measured are cliffs, trees, and buildings. The members could visit a bank or a factory, and they would find some benefit in simply going on a sightseeing tour. On a tour they could look for mathematical forms in many familiar objects—the various angular and curvilinear designs in architecture and the symmetry to be found in nature.

A source of fascinating program material is to be found in the multitude of mathematical problems in which an apparently correct chain of operations leads to an absurd result. These problems are known as mathematical fallacies. Children of high school standing obtain a greater thrill out of "proving" a theorem that cannot possibly be true than

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[10]Hoag, Ruth: "Program Material and Some Types of Program Work Which Might Be Undertaken by High School Mathematics Clubs," *The Mathematics Teacher*, Vol. XXIV (Dec., 1931), pp. 492–502.

[11]Hoag, *op. cit.*, pp. 492–502.

they usually do out of working the rigid and logical demonstrations of the theorems of geometry, and the hope of detecting some discrepancy in the proof prompts the pupils to wrestle with intricate points which would have seemed commonplace in other situations. It has been found in some cases that if the sponsor remains in the background sufficiently, placing the responsibility of deciding upon the validity of the proofs presented and of accepting or rejecting the solutions given upon the members of the club, that the members will become more alert and adopt a questioning attitude which will result in animated discussions and arguments long past the usual time for adjournment over the solution of a particularly difficult problem.[12] The following theorems are some of the better known theorems falling into the category of mathematical fallacies:

> Any point on a line segment is the midpoint of the segment.
>
> Two triangles are congruent when any two sides and an angle of one are equal to the corresponding parts of the second triangle.
>
> The semicircumference of a circle equals its diameter.
>
> An obtuse angle is equal to a right angle.
>
> More than one distinct perpendicular may be constructed from a point without a given line to that line.
>
> All triangles are isosceles.
>
> Sixty-five equals sixty-four.

It would be well to include some geometric puns, for they serve as an interesting topic for presentation in dialogue form or in some form that circumstances will permit because the students have heard puns over the radio during the programs of such radio comedians as Ed Wynn and Jack Pearl. The following puns were taken from Zulu Reed's article, "High School Mathematics Clubs," appearing in the October, 1925, issue of *The Mathematics Teacher*, Vol. XVIII, p. 355:

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[12]Shoesmith, Beulah I.: "Mathematics Clubs in Secondary Schools," *School Science and Mathematics*, Vol. XVI (Feb., 1916), p. 109.

The Pun	*Its Meaning*
A flattering remark	A complement
That which Noah built	An arc
A harmless kind of worm	An angle
A special kind of meter	A diameter
What the bloodhounds do in chasing a woman criminal	They center
A part of John the Baptist's diet	Locus
A gram, but not a measure of weight	A parallelogram
A tall pot in use	A hypotenuse
An appropriate title for a knight whose name is Kell	Circle

The appeal to the competitive instinct is one of the strongest appeals that may be made to the interest of the pupil. They want their group to win because it is their group, and they desire to be known as one who had a part in the affair. We may find several desirable results ensuing from the use of games and contests in the classroom and in the club room. Usually games and contests arouse considerable interest and give rise to many pleasurable minutes. The logical by-product would seem to be that some aspect of mathematics impresses itself deeply on the unconscious minds of the participants. Proponents of the new doctrine of play in education advance the theory that the capitalization of the competitive and emulative instincts tends to foster a desirable social spirit which manifests itself in the subordination of one's own desires to the good of the group by unselfish and honorable dealing and by the exhibition of kindness to one's opponents.

There are two general types of games, those involving a chance element and those involving skill and quickness of some kind.[13] In the first class we have "Old Maid in Algebra."[14] This game is similar to the Old Maid game in cards. In this game the cards are prepared in pairs. The basis of pairing may be a common root to equations appearing on

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[13]Drummond, Margaret: *The Psychology of Teaching of Number*, p. 77.

[14]Raster, Alfreda: "Mathematical Games," *The Mathematics Teacher*, Vol. XVII (Nov., 1924), p. 422.

two of the cards, as if one card had the equation $2x = 6$, while the other card had the equation $3x = 9$. These cards match, for the equations appearing on them possess the same root, $x = 3$. There is an odd card. One player holds up his cards in fan-like manner with the backs of the cards to the one next to him. This person draws a card from the cards held up and tries to match it with one of his own. Then he holds up his cards for the player next to him to draw from, and the one matching all his cards first is the winner, while the one who holds the odd card is termed the Old Maid. Fractions having a common value when simplified, or names of expressions and the expressions themselves may be used as bases for matching.

There are a multitude of games and contests based upon some particular skill. While the contests may reach inter-school proportions, we shall confine ourselves to a discussion of those games suitable for a club program. It may be mentioned at the start that these games and contests need not be restricted to club programs alone, for much benefit may be gotten from a mathematical game in the classroom, as it affords a fascination manner of review, in addition to affording a pleasant diversion from the more formal work of the classroom. Teachers wishing to effect a review of mathematical terminology, for instance, may start a spelling contest in which the words used are words from a list of mathematical terms. Another form of contest is known as a Crossed Words Contest,[15] a contest in which a list of mathematical terms is prepared, but with the spelling completely scrambled. The first student to unscramble the words is the winner. To give some hint of the possibilities of such a contest, we may include in our list such letter groups as noitequa, rtcfoa, rpgha, which when unscrambled, give respectively equation, factor, and graph.

Also we might include mathematical word puzzles, examples of which may be found in volumes twenty-five and twenty-six of *School Science and Mathematics*. With a comparatively small amount of effort the teachers or sponsors

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[15]Raster, *op. cit.*, p. 424.

could devise a cross word puzzle that would bring about the desired results. The pupils would enjoy working them and would enjoy devising some of the puzzles themselves.

Closely associated with the idea of mathematical vocabulary games is the contest described by Helen M. Walker in her article, "A Mathematics Contest," appearing in *The Mathematics Teacher,* Vol. XX, May, 1927, page 274. As each guest to the mathematics club entered the room he was given a sheet of paper bearing the numbers from one to forty-two with the instructions: "Distributed about the room is a mathematics exhibit, each article of which represents a familiar term taken from the field of elementary mathematics. Each article in the exhibit is numbered. Write down the term represented by an article opposite the corresponding number on the paper provided." Included in the exhibits was a drawing of a boat, representing an arc; pine cones, representing cones; a picture of an airplane, representing a plane; and a picture of a monkey, representing evolution.

Another game consists of giving one group of students papers with equations upon them and giving another group of students papers bearing the solutions to these equations. Each student with an equation is to find the person having the paper bearing the solution to this equation.

Allow me to appropriate a few lines to the description of a game especially appealing to all sport-loving pupils. The following game is called an Algebra Baseball Game and was taken from the April, 1930, issue of *The Mathematics Teacher.* The following rules are used in playing the game:

1. The clubroom shall be the diamond or ball park.
2. The teams shall be chosen from among the members.
3. The baseballs shall be the problems which the pitcher is to pitch to his opponents.
4. The following officers shall rule the game: the timekeeper, the umpire, and the scorekeeper.
5. The game shall be composed of innings, three outs on both sides constituting an inning.
6. The pitcher shall have one minute in which to pitch the ball.
7. The batter shall then copy the problem on the board and proceed to solve it if he can.

8. A "ball" shall be called against the pitcher when a problem has not been pitched before the expiration of one minute or when he shall have given the same problem twice.

9. A "strike" shall be called with every pitch.

10. If the batter solves the third strike before the catcher shall have solved it, he advances to the bases according to the difficulty of the problem, a simple problem advancing him to first base, a somewhat harder problem advancing him to the second base or third base or even to home plate.

11. A batter shall be out when he fails to solve a problem.

12. A runner may be put out on the bases if he shall fail to solve a problem given by the pitcher in an attempt to catch the runner "napping" off a certain base; however, the runner shall score when he has been forced around the bases or when he shall have solved a sufficiently hard problem.

Let us consider briefly the part played by puzzle problems. The idea of mathematical recreations is not a new idea by any means. The human mind has always found pleasure in puzzles and in tricks of all sorts. Cantor attributes the first mathematical puzzle to Ahmes, about 2000 B.C. The problem of the goose, the fox, and the bag of corn and how they were to be taken across the river was known to Alcuin in the time of Charlemagne. Magic squares were known to the Hindus and to the Arabs very early, and a magic square problem appeared in a Chinese book as early as 1125. The problem of the hare and the hound has been found in the Italian arithmetics of about 1460. Early books dealing with recreational problems were *The Whetstone of Witte,* published by Recorde in 1557, and *Propositiones ad acuendos iuvenes,* written by Alcuin (735–804).[16]

It is hardly necessary to delve into the various phases of recreational material such as unusual problems, magic squares, and magic circles, for there have been some excellent books published containing many of them. The following short bibliography will prove to be beneficial:

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[16]Newhall, Charles W.: "Recreations in Secondary School Mathematics," *School Science and Mathematics,* Vol. XV (April, 1915), p. 278.

Ball, W. W. R.: *Mathematical Recreations and Essays*, The Macmillan Company, New York.
Dudeney, H. E.: *Amusements in Mathematics*, Thomas Nelson and Sons, New York.
Jones, S. I.: *Mathematical Wrinkles*, Nashville, Tenn., Author.
Licks, H. E.: *Recreations in Mathematics*, D. Van Nostrand and Company, New York.
White, W. F.: *Scrapbook of Elementary Mathematics*, Open Court Publishing Company, Chicago.

There are many children who like to act and who would become intensely interested in mathematics if they could apply it to drama. Many teachers have become cognizant of this fact and have written plays based upon mathematics. The following list of plays, accompanied by a notation as to where they may be found, will be suggestive:

"Alice in Numberland," Hedges, Blanche B., *The Mathematics Teacher*, Vol. XXVI (April, 1933), pp. 222–226.
"Alice in the Wonderland of Mathematics," White, W. F.: *Scrapbook of Elementary Mathematics*, pp. 218 ff.
"A Little Journey to the Land of Mathematics," Crawford, Alma E., *The Mathematics Teacher*, Vol. XVII (Oct., 1924), pp. 336–342.
"A Living Theorem," Hatcher, Frances B., *School Science and Mathematics*, Vol. XVI (Jan., 1916), pp. 39–40.
"A Mathematical Victory," Harding, P. H., *School Science and Mathematics*, Vol. XVII (June, 1917), pp. 475–482.
"A Near Tragedy," Miller, Florence Brooks, *The Mathematics Teacher*, Vol. XXII (Dec., 1929), pp. 472–481.
"An Idea that Paid," Miller, Florence Brooks, *The Mathematics Teacher*, Vol. XXV (Dec., 1932), pp. 470–479.
"Case of Matthew Mattix," Smith, Alice A., *The Mathematics Teacher*, Vol. XXVI (Sept., 1933), pp. 286–291.
"Evolution of Numbers, A Historical Drama," Slaught, H. E., *The Mathematics Teacher*, Vol. XXI (Oct., 1928), pp. 305–315.
"Falling in Love with Plain Geometry," Hatton, Caroline, and Smith, Doris H., *The Mathematics Teacher*, Vol. XX (Nov., 1927), pp. 389–402.
"Flatland," Anonymous, *School Science and Mathematics*, Vol. XIV (Oct., 1914), pp. 583–587.
"Geometry Humanized," Scott, Erma, *The Mathematics Teacher*, Vol. XXI (Feb., 1928), pp. 92–101.
"If," Snyder, Ruth L., *The Mathematics Teacher*, Vol. XXI (Dec., 1929), pp. 482–486.
"Mathematical Nightmare," Skerrett, Josephine, *The Mathematics Teacher*, Vol. XXI (Nov., 1929), pp. 413–417.

"Mathesis," Brownell, Ella, *The Mathematics Teacher*, Vol. XX (Dec., 1927), pp. 459–465.
"Math Quest," Whitaker, Helen, *The Mathematics Teacher*, Vol. XVIII (Oct., 1925), pp. 356 ff.
"Mock Trial of B versus A," adapted by Kathryn McSorley from Stephen Leacock's story, "A. B. C.," *School Science and Mathematics*, Vol. XVIII (Oct., 1918), pp. 611–621.
"Number Play in Three Acts," Schlierholz, Tillie, *The Mathematics Teacher*, Vol. XVII (March, 1924), pp. 154–169.
"Point College," Schlaugh, Helen M., *School Science and Mathematics*, Vol. XXXI (April, 1931), pp. 448–454.
"Socrates Teaches Mathematics," Anning, Norman, *School Science and Mathematics*, Vol. XXIII (June, 1923), pp. 581–584.
"The Mathematics Club Meets," Pitcher, Wilemina Everett, *The Mathematics Teacher*, Vol. XXIV (April, 1931), pp. 197–207.

An interesting diversion for poetry and music lovers is afforded in a mathematics club meeting by letting them present their own poems or songs to the group. It is good psychology to keep the members happy, and this can be accomplished to a great extent by having club song services. The following poems will serve as examples for the students who desire to write poems:

"A Word to the Foolish," Fezandie, Margaret, *The Mathematics Teacher*, Vol. XIX (Feb., 1926), p. 100.
"First Aid in Algebraic Fractions," unknown, *The Mathematics Teacher*, Vol. XIX (Feb., 1926), p. 101.
"Love Mathematical," Gillies, Captain Robert C., *The Mathematics Teacher*, Vol. XII (March, 1920), p. 124.
"Ode in Praise of Mathematics," unknown, *The Mathematics Teacher*, Vol. XIX (Dec., 1926), p. 487.
"So Let Me Work," Merrill, Helen A., *The Mathematics Teacher*, Vol. XIX (Feb., 1926), p. 99.
"Tangents," Williams, Glen, *The Mathematics Teacher*, Vol. XIX (Feb., 1926), p. 99.
"Ode to a Cosine," Williams, Glen, *The Mathematics Teacher*, Vol. XIX (Dec., 1926), p. 488.

The following songs may be sung at the club meetings as a means of getting "started right":

"If There Were No Mathematics at All." Tune: "If I Had a Talking Picture," *School Science and Mathematics*, Vol. XXX (July, 1930), p. 634.

"Number Song." Tune: "Lulu Is Our Darling Pride," *The Mathematics Teacher*, Vol. IX (June, 1917), p. 208.
"Conic Sections." Tune: "Tune of Mistress Shady," *The Mathematics Teacher*, Vol. XXV (June, 1932), p. 36.
"Sing a Song of Six Points." Tune: "Sing a Song of Sixpence," *The Mathematics Teacher*, Vol. XXV (June, 1932), p. 37.
"Greek and Mathematics." Tune: "Yankee Doodle," *The Mathematics Teacher*, Vol. XXV (June, 1932), p. 37.

Now that we have a general idea of what may go to make up a mathematics program, let us note some programs that have been organized and presented:[17]

Program One:
1. Magic Squares and their Construction.
2. Magic Squares in Design.
3. Curious Problems.
4. Division Problems.
5. A Playlet, "Geometry Humanized."

Program Two:
1. Development of Algebraic Symbols.
2. Interesting Stories from Mathematical History.
3. Sketch, "Meeting of the Mathematics Association for the Purpose of Improving Conditions in the High School."
4. Mathematics in Art and Architecture.

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[17]Taylor, Helen S.: "A Mathematics Library and Recreational Programs, *School Science and Mathematics*, Vol. XXX (July, 1930), p. 626.

THE FUNCTION OF THE TEACHERS COLLEGE IN EXPERIMENTING IN CURRICULUM REVISION IN MATHEMATICS FROM THE VIEWPOINT OF THE UNIVERSITY*

BY H. J. ETTLINGER
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It is a pleasure and a privilege to be invited to present a paper before the Fifth Annual Teacher Training Conference at the North Texas State Teachers College. The subject assigned to me, "The Function of the Teachers College in Experimenting in Curriculum Revision in Mathematics from the Viewpoint of the University," presents many angles which are of profound interest. It may be said that in a large measure the problem of curriculum revision is co-extensive with that of the complete field of education, and it is apparent that a teachers college must be profoundly concerned with this problem. I take it that the good teacher, the teacher who is vitally concerned with his students and with his subject, is one who is constantly revising his own presentation and point of view, and the material of the courses which he teaches. Hence, it is not difficult to arouse one's enthusiasm for the particular problem involved in curriculum revision with respect to mathematics.

I am not sure that my address will concern itself in a particular way with the first part of the title assigned, nor will we overdo the last part either. The teachers college has achieved a definite place in our state educational system, and in particular it must furnish as expertly trained teachers as it is possible to produce. On the other hand I do not know that the university's point of view has many special features with respect to mathematics in the secondary schools that are peculiar to itself. May I say that in the large we are all component members of an organization which is endeavoring to raise the level of life and living,

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*Read before the Fifth Annual Teacher Training Conference, North Texas State Teachers College, Denton, Texas, March 16, 1935.

and general understanding of the world in which we live. We are very zealous in our task of turning out as efficiently as possible young people who will take their places as useful and intelligent citizens. We hope that they will take their duties, obligations, and responsibilities as seriously as possible, but at the same time with a maximum of happiness and enjoyment. I have yet to find anyone who will argue that in promoting these ends the fullest possible development of the number sense is not desirable. On the other hand, from many points of view one is able to show the definite place of training in numbers in our educational curriculum.

The chief difficulty has been that misconception and misunderstanding have arisen with respect to what constitutes mathematics. How many people today, in fact, how many elementary teachers of mathematics, realize that all of the developments of mathematics from the lowest to the highest are extensions of the very simplest numerical process—that of counting? One must emphasize at this point that the very great importance of the very simplest operation with numbers, namely, the count, is not realized by teachers of mathematics as a rule. There are a great many stimulating and simple situations which can be utilized in this respect, for example, counting in other units by terms such as dozens, gross, and great gross; quires and reams; seconds, minutes, and hours. In the *Atlantic Monthly* of last October there is an eight-page article by a non-mathematician on counting by dozens which is of exceedingly great interest. The writer is the Director of Publications for the Russell Sage Foundation. The counting process is not merely of aesthetic interest, but is also practically important both in every-day life and in scientific applications. Finally, I must mention its very great importance in the farthest reaches of what is called higher mathematics.

Those who prepare teachers of mathematics must saturate such prospective teachers with numbers as completely as possible. Teachers colleges and universities must join in doing this. This is but another way of saying that the more deeply grounded in the entire field of mathematics the student is the better teacher he will make in mathematics at even the lowest elementary level. Counting numbers, positive integers, negative numbers, fractions, irrational numbers, algebraic numbers, transcendental numbers, must course through the mind of the teacher even though the field presented to the elementary student is much narrower. The number scale leads in various directions to help the teacher ground his students deeply in the number concept.

With a background such as I have just described, no elementary class in mathematics will become lost in the mysticism of algebraic symbolism as they usually do at the present time. If the student deals with numbers and works with numbers before endeavoring to acquire some facility in the language of numbers, which is ordinarily called algebra, he will not find himself completely lost in a strange and forbidding land. It is true that later he will be exercised and drilled so that he may acquire a sufficient amount of technique both to understand the language and to make use of it, or at least to be able to translate it into a simpler terminology than the usual technical one. There is great need for simplifying the nomenclature at present in use in such subjects as algebra, plane geometry, and trigonometry so that those difficulties which are due merely to the use of specialized terminology may be eliminated. Furthermore, it must be recognized that there should not be an overemphasis upon the mechanical side of manipulation or drill work for the reason that the amount of skill to be developed in the average student cannot be set too high and, for the best student, it is boresome and unnecessary.

The contents of the every-day life of the student should be utilized to the fullest in providing atmosphere and background for his number work. The reasons for this are many. In the first place, one is more likely in this way to talk of things in terms of situations within the experience of the student. One need not go far afield to do this inasmuch as automobiles, airplanes, and radios are well within the experience of everyone, but this endeavor to do what is called "vitalizing" the subject must not go too far. When one brings in scientific situations that are outside the experience of the student one may hinder fearfully instead of helping him; nor in endeavoring to give concrete examples must applications be overworked. There is some danger that the best talents will be submerged by this process. I recall a criticism once expressed by a student who had been given twenty applications of a certain theorem to do; he remarked that if you changed one number in each of the applications they would all be alike.

At a very early stage in mathematics graphical methods and diagrams should be used to the fullest. Pictorial writing came very early in the intellectual history of the human race and pictorial representation in the early history of the number experience of the individual can be most helpful. The number scale mentioned above is a beginning, to be followed later by diagrams and simple bar graphs of various kinds. Certainly there is no person who can dodge the responsibility of interpreting numbers which are presented to him (in the way of civil or business reports) in some diagrammatic form. This is also true with respect to the relation between numbers and the most advanced methods of geometric developments connected with number relations.

The main concern of this paper has been with the orientation and the atmosphere which should be present in connection with the teaching of elementary mathematics. No specific recommendation is made as to time or content of courses for the secondary schools; that belongs at a later stage than our present curriculum program has attained. I find myself completely committed to the experimental point of view in mathematics, and in the hands of a selected group of competent teachers I would favor experimentation with materials and points of view. In fact I believe that mathematics itself should be approached by the young student as an experimental science rather than as an exact science. Not that the latter aspect should be completely neglected, because even an experimenter when he wishes to determine the success of his experiments must check and recheck and endeavor to give logical reasons for his conclusions. In plane geometry, for example, far too much emphasis is placed on demonstration, which should be replaced by some work connected with drawing and construction as well as experimentation with new geometric figures.

Finally, may I make a plea for consideration for the better students in the class, as well as for the average students? The good teacher of mathematics, like all good teachers of any subject, must realize that democracy in education does not necessarily mean that a dead level should be maintained in the class room. The needs of the better students must be taken care of as well as the needs of the average students and those of the less able ones. In order to keep the subject matter as growing and alive as possible, it is necessary to stimulate the better students by placing before them the possibility of extra problems and of actually presenting in class ideas that may go over the heads of the remainder of the class. A proper balance with respect to this point of view will not sacrifice the interest of the other students, but will at the same time give rise to the development of the best talent which the teacher may meet in his class. The State of Texas has potential mathematical resources in its native sons and daughters which are equal to those to be found anywhere in the world. To neglect the development of these would be just as reprehensible as to fail to develop agricultural or industrial possibilities.

A COMPARATIVE STUDY OF THE TEACHING OF PLANE GEOMETRY IN THE UNITED STATES AND FRANCE*

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Types of Schools in France

In order to make a satisfactory comparison of the teaching of geometry in the United States and France, it is desirable to make a comparative observation of the systems of education in the two countries. It will, of course, be assumed that the reader is already familiar with the fundamental facts regarding education in the United States.

There are three degrees of education in France: namely, primary, secondary, and higher. These do not correspond exactly to the divisions in the United States under the same notation. In this country the so-called primary and secondary divisions are superimposed one upon the other, while in France they are to a certain extent concurrent. Their primary system extends into what we call the secondary field, and their secondary system extends back into what we call the primary field, and parallels it, although with certain modifications.

There are three general types of school in France, corresponding to the three degrees of education mentioned above. These are: (1) L'École Primaire, or Primary School; (2) Lycées, Communal Colleges, and private schools, comprising the secondary schools; (3) the universities and other higher institutions. The Primary School is composed of the infant schools, the elementary primary school, and the upper primary school. The infant schools, like our kindergartens, have no real connection with the school system as a whole, but are merely preparatory to entrance into the school proper.

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*Adapted from his M.A. thesis of a similar title, The University of Texas, June, 1934.

Attendance is compulsory for the ages from five to thirteen inclusive, and it is free for this period of time. Thereafter it is only partly free. The infant school takes care of pupils of ages from two to six; the elementary primary school provides for the ages from six to thirteen; and the upper primary school, for the ages from thirteen up to fifteen or sixteen. It will be noted here that the upper primary school runs concurrently with the lycée during a portion of the school age, covering about the ages from thirteen to sixteen.

The aim of the Primary School is "to develop the judgment and train the reasoning faculties." The subjects generally taught in the courses are morals, reading-writing-arithmetic, natural science, history and geography, drawing, manual training, singing, and physical culture. The presentation of the course is practical, rather than theoretical; it purposes to enable the child to learn by doing and experimenting, rather than by merely observing.

The lycée and collège communal differ mainly in the manner in which they are supported, the former being entirely under the support of the state and the latter being supported partly by the state and partly by the commune, or municipality. Also, the lycée requires a higher standard than does the collège. It is decidedly a school for the élite, and is patronized by the middle class citizen.

Secondary education in France forms a complete unit in itself, from the preparatory stage to the university, whereas in the United States secondary education is a continuation of the elementary school. One purpose of the école primaire supérieure, or upper primary school, in France is to bridge the gap between the primary and the higher institutions. France regards an intellectual proletariat as dangerous, and wants only a practical education for the masses. Skilled industry is provided for, but a training which is purely technical is not desired. Such training is given by a different type of school called l'école professionnelle, or professional school. In other words, there must be for the masses a general culture within certain limits.

French and American Examinations Compared

Secondary education in France is generally thought of as that furnished by the lycée and its parallel school, the collège. This is roughly equivalent to the American junior and senior high schools combined, if we omit the Special Class at the close of the lycée. The years, or "grades," in the French secondary school are designated as follows: Sixth Class (or Form), Fifth Class, Fourth Class, Third Class, Second Class, First Class, after which comes the Special Class, this being either the Class of Philosophy or the Class of Mathematics.

The first important goal of the student in the secondary school is the examination called the baccalauréat. This is divided into two parts, the first of which is taken when he finishes the first class. He then studies for a year in a special class of either mathematics or philosophy, after which he takes the second part. These examinations are both written and oral. The first part of the baccalauréat may be compared in a general way to the College Entrance Board Examinations in our own country, but the two examinations do not cover the same requirements. In mathematics, the algebra in the first part is somewhat more advanced than our first year algebra in the high school. Their geometry is equivalent to our advanced requirements in geometry; the French school correlates geometry with other subjects more than we do, and places more stress on calculations. In the second part of the baccalauréat, their algebra course covers all the essential material, but not as much as is required in this country. The second part is composed mainly of material in analytic geometry, while in this country this subject is generally given only in college, and often not until the second year of college. Geometry is stressed in both parts of the examination more than the other branches of mathematics, and especially is stress placed upon the applications of geometry. In the United States geometry and algebra are about equally stressed, and the applications of these subjects are generally neglected. This is partly due to the fact that we do not require of all students a knowledge of physics to the degree that it is required in France. The French students must attain a greater degree of efficiency and ability to use the mathematics that they have learned than is required in most of the schools of America.[1]

Trends in Teaching Mathematics in France

By the Reform of 1923 the course in mathematics was made uniform for all students from the sixth to the first class. Prior to this reform the program was arranged for specialization to begin early in the curriculum; since then it begins in the first class. There are some who are not mathematics-minded, and it is difficult for them to pursue the study of mathematics with those who are more fortunate, but it is thought that, by selecting subject matter of average difficulty and postponing the more complex matter to the special mathematics course, all pupils can reach the first class. The purpose of the course up to this point is to train the reasoning powers, and not to specialize. It is noticeable here that the French do not place any special emphasis on mathematics during the first five years of the lycée; French education is of the linguistic, literary type. However, this apparent neglect of mathematics is offset by the fact that intense concentration takes place when specialization begins. The lack of emphasis on mathematics is evident again when we consider that up to the time of taking the special course in mathematics the pupil does not cover more than algebra and plane and solid geometry.

An important characteristic of the French teaching of mathematics is the close relation that is found among the various mathematical subjects. In our schools the teaching of arithmetic is conducted up to a certain year, generally the first year of high school, and is there succeeded by algebra, which in its turn is succeeded by geometry. The French method is to carry concurrently at least two subjects in mathematics from the fifth class in one division and from

[1]Cabot, S. P.: Harvard Bulletins in Education, No. 15, p. 36.

the fourth in the other. In this way some of the difficulties in one subject are avoided by means of the elementary notions obtained through the study of some other mathematical subject. Geometry is made more real by emphasizing its arithmetical relations. Even in the infant school a few geometric principles are learned by sense training, assisted by suitable apparatus. In the elementary school geometry is still parallel to drawing. The tendency is to develop the intuitive and visual side of the subject so as to train the observational and reasoning powers of the children. Mechanical drawing, in certain divisions, brings into play the knowledge gained in all the subjects. As a result, "mathematics work appears as a single unified subject with several facets rather than as so many discrete studies of the school curriculum."[2]

Before the Reform of 1905 geometry had been presented in the manner of the Elements of Euclid, but since then it is to be taught in the first four classes in an experimental manner. The presentation of matter pertaining to motions of lines, planes, parallels, etc., is to be accompanied by figures showing their various positions. In the first four classes the course in geometry is to be as brief and as concrete as possible, but in the second and first classes the subject is to be resumed for a logical study in the scientific classes, with plane geometry in the second class and geometry of space in the first.[3]

Since textbooks in France must be compiled so as to conform to the aims and objectives of teaching in the nation as a whole, statements from the authors of these books are significant. In one of the plane geometry texts of recent publication the author says in substance the following: The greater part of the students in the French secondary schools, and even the best of them, find serious difficulty in the solution of problems in geometry. They are bewildered by such terms as theorem, axiom, postulate. As they

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[2]Farrington, F. E.: *French Secondary Schools*, p. 271.

[3]Bioche, M. Ch.: Commission Internationale de L'Enseignement Mathématique, Sous-Commission Francaise, (A) *Rapport*, p. 5.

struggle with the proofs of theorems, it is with the impression that certain truths have always existed, having been discovered little by little and then announced to the world. The author plans in this text to persuade the student, by exciting his curiosity, to find out new truths for himself. The fundamental principles are experimental, and the experiments are to be made by the pupil, through the use of tools and devices, such as a visiting card, and some pins, a silk thread, etc., in order that the course may "be based upon tangible realities and not upon abstractions that the student cannot understand." It is expected that the intuition and experiences of the student will furnish him with responses to the first questions that arise. The methods used will appeal to his intelligence, and not to his memory. The author's appeal is in keeping with the opinion on the part of some educators that geometry has its chief value in developing certain reasoning powers.

In conducting this investigation a number of modern French textbooks in geometry have been examined, but only three have been examined with any degree of thoroughness. They are:

Chenevier, Pierre: *Précis de Géométrie Plane* (1932), for use in the Fourth and the Third Classes.

Chenevier, Pierre: *Cours de Géométrie* (1930), for use in the Second and the First Classes.

Chenard, H.: *Géométrie* (1924), for use in the industrial schools, first year.

A Comparison of Methods in the Two Countries

In Chenevier's *Précis de Géométrie,* which is used in the first year of formal geometry, we find the following classification of subject matter:

PART I

STRAIGHT LINE AND CIRCLE

Chapter I: Preliminary Notions of the Straight Line
Chapter II: Angles
Chapter III: Circles. Measure of Angles
Chapter IV: Polygons.—Equal Figures, Isosceles Triangles

Chapter V: Perpendicular and Oblique Intersection of a Line and of a Circle. Geometric Constructions
Chapter VI: Parallels
Chapter VII: Determination of a Circle. Relative Positions of two Circles. Chords and Arcs
Chapter VIII: Angles Whose Sides Are Parallel or Perpendicular
Chapter IX: Construction of Triangles. Case of Equality. Applications
Chapter X: Parallelograms
Chapter XI: Measure of Arcs and Angles. The Inscriptible Quadrilateral
Chapter XII: Problems on the Tangent and Circle
Chapter XIII: Concurrent Lines in a Triangle

PART II

SIMILITUDE, AREAS

Chapter XIV: Proportional Segments
Chapter XV: Similar Triangles
Chapter XVI: Applications of Similitude of Triangles
Chapter XVII: Regular Polygons. The Perimeter of a Circle
Chapter XVIII: Areas of Polygons

There are, on the average, about twenty-five exercises at the end of each chapter. Many of these are practical applications of the content of the particular chapter.

In his more advanced text, *Cours de Géométrie*, the same author uses a scheme of classification very similar to the one just shown. However, the first part of this text covers plane geometry, and the second part, geometry of space. As the author sets out in the preface, the *Cours de Géométrie* is adapted to meet the requirements of the students who have studied plane geometry for two years. Besides a great deal of new material, it includes a repetition of the fundamental material of the first course.

Chenard's *Géométrie* was compiled for use in the industrial schools and the industrial sections of the regular schools. The title page bears the statement that the text is published under the direction of Felix Martel, the Inspector General of Public Instruction. This statement is important in that it indicates that, instead of the state trying to select the most suitable text from a number already published, as is generally done in this country, the publishers are expected to produce texts that meet certain established aims set up by a centralized bureau of education.

This text is intended for use in the first-year course of elementary geometry. The arrangement of subject matter is as follows:

Chapter I: The Line and the Plane
Chapter II: Generalities on Displacements
Chapter III: Rotation and Derived Figures
Chapter IV: Perpendicular Figures
Chapter V: Rectilinear Translation. Parallelism
Chapter VI: Symmetry
Chapter VII: Polygons
Chapter VIII: Notion of Distance. Angle of a Line and of a Plane
Chapter IX: Circumference

In general the methods of procedure in the French texts are similar to those found in the current texts of our country, but there are many interesting points of difference. In the former, numerous devices are used in order to make the study more enlivening. For instance, a straight line is represented by sticking two pins into a wooden board or cardboard and drawing a black thread tightly around the pins. Furthermore, if a red thread is drawn in the same manner around the pins, placing it exactly upon the black thread, we have roughly the notion of the coincidence of lines. This also illustrates the familiar postulate: Through two points there can be passed one, and only one, straight line. This is given as the fundamental property of the straight line. By the same method of reasoning it is shown that two straight lines which have two common points coincide.

A plane is defined as a surface such that every straight line which joins any two points of this surface is wholly contained in the surface. To illustrate the definition a ruler is applied to the top of a smooth table. If the edge of the ruler rests entirely upon the surface of the table, with all points of the edge touching the table, the surface of the table is plane. The edge of the ruler is wholly upon the table if two points of the ruler are upon it.

A visiting card placed flat upon the table and moved around illustrates the gliding of one plane upon another with which it coincides. In any position all the points of the card coincide with points of the table. This notion may be stated briefly as follows: *A plane may be made to glide upon itself.* If the card is turned over, the operation is known as "retournement," and the above statement is still true. Then it may be said that two planes may be made to coincide without retournement, or with retournement.

Such devices are insignificant in themselves, but they serve to illustrate in a very vivid way certain methods of demonstration so prevalent in the French texts.

Let us now compare some methods of approach to propositions found in the commonly used texts of the two countries.

A very familiar fundamental theorem is: *Two vertical angles are equal.* The usual method in the American schools is to state the theorem first and then require the student to prove it. In the French texts under examination the student is taught to find out certain truths from a given hypothesis, and finally to state the theorem as a conclusion. The theorem is not stated in the beginning. The only important remark given by a certain author in the presentation of the above theorem is, "We have just seen that c and d are supplements of the same angle."

Another example of French procedure is introduced by the question, "What relation have the bisectors of two adjacent supplementary angles?" The pupil is led to discover that the angle formed by the two bisectors is equal to one-half the sum of the two supplementary angles, and finally that they are perpendicular. The theorem is then stated as follows:

Theorem.—The bisectors of two adjacent supplementary angles are perpendicular.

In a similar way an approach is made to the theorem which states, in substance, that only one perpendicular can be drawn to a line from a point outside that line. The discussion is interesting from the fact that use is made of the drawing square, which contains a right angle. A ruler is so placed that its edge coincides with the given line. With one side against the edge of the ruler, the square is made to glide along the edge of the ruler until the other side passes through the given outside point. Then a line is drawn through the point along the side of the square and prolonged beyond the given line. This perpendicular cuts the given line at a certain point, and another point is placed on the line. The paper is now folded along the given line and a point corresponding to the given point is located on the opposite side of the given line. The corresponding elements on opposite sides of the line can be made to coincide. The remainder of the demonstration is similar to that found in our own texts. The theorem is finally stated.

Another reason for concluding that only one perpendicular can be drawn, in the above case, is the fact that a line can be drawn through the given point along the edge of the square in only one way.

It is evident that in some of the proofs in the French texts complete rigor is lacking. This conforms to the opinion of some mathematicians that many statements in geometry should be taken as true intuitively and with little or no proof.

In one of the preceding demonstrations it will be recalled that the conception of symmetry was applied. There are two other notions that are used a great deal in the French texts, but not generally used in our texts. These are *rotations* and *translations*, which are merely two different forms of displacement.

We shall give three general assumptions which the French authors make in regard to displacements:

(1) Since a geometric figure is a conception of the mind, it is immaterial by nature. Hence a change of position is fictitious.

(2) When a geometric figure is displaced its rigidity is preserved; that is, its elements are not changed in their relation to one another.

(3) The element of time does not intervene in the displacement of a geometric figure.

Considering movements both in a plane and in space, rotation may be defined as a movement of a line about a point regarded as a pivot, or the movement of a line or a plane about a line regarded as an axis.

The translations considered in these texts are mainly of the rectilinear type. Such a translation is defined as a movement in which a line segment of a figure glides along the straight line of which it is a part, the rigidity of the figure being maintained throughout the movement.

The movement of rotation is applied to the circle, and is compared to the turning of a cart wheel in the air. The student is directed to procure a visiting card, place it on a sheet of paper on a drawing board, and pierce it with a pin, fixing it to the board at a certain point, say *O*. The card is now rotated about the pin, while the sheet of paper remains at rest. Like the cart wheel, the visiting card is animated by a movement of rotation. As we are studying only that which takes place in the plane, we say that this rotation takes place around the point *O*.[4]

Now suppose that with this same device we pierce a little hole through a point other than *O*, say *A*, just large enough to engage the point of a pencil. As we rotate the card around the point *O* the distance *OA* remains constant, and the point of the pencil describes a circle of center *O* and radius *OA*. Any other point of the card will also describe a circle of center *O*. Since the card describes a movable plane which turns about the center *O*, we come to the following fundamental property:

> In a movement of rotation about a point, all the points of the movable plane describe circles having for a center the fixed point.

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[4]Chenevier, P.: *Précis de Géométrie Plane*, p. 40.

We shall now illustrate the meaning of *displacement* in a rotation. Let us take again the visiting card fastened to the sheet of paper with a pin. Let us place it in motion, then bring it to rest. We have now displaced the card. As this displacement results from a movement of rotation, we say that the card has undergone a displacement by rotation. The displacement is characterized by an initial position and a final position. Furthermore, if the card and the sheet of paper are thought of as coinciding, we may say that a plane may be made to rotate upon itself. We find this operation used a great deal in the French demonstrations.

In a similar manner it may be said that a circle can be made to glide upon itself. For if we trace upon a visiting card and upon a sheet of paper two circles of the same radius, and then stick a pin through the two centers with the one upon the other, our circles coincide during the entire movement of rotation of the card around the common center of the two circles. The conclusion may be stated as follows:

> Theorem.—A rotation around its center makes a circle glide upon itself.

The idea of rotation is used in the discussion of circles in dealing with the central angle. For instance, as the point A mentioned above moves about O it describes a circle of center O. During the movement of the radius it always continues to pass through the fixed point O. Any two positions of the radius OA may be taken as the sides of an angle whose vertex lies on the center O. Such an angle is called a central angle. It is shown also that a central angle cuts off on the circle an arc of a circle, and conversely, that to every arc of a circle we can make correspond a central angle.

The equality of two arcs of a circle may now be determined by the method of rotation. It is assumed that the arcs are equal if they can be made to coincide. If the arcs are taken on the same circle, we can make them coincide by causing a displacement of one of them by rotation around the center of the circle, in such a manner that one arc is brought upon the other. If the arcs are taken upon equal circles, the two circles must be made to coincide by placing the circles together with the center of one upon the center of the other, after which the procedure is the same as before.

The French texts which we are examining consider two classes of congruent figures:

1. Figures directly congruent, which may be made to coincide by a displacement without turning over.
2. Figures inversely congruent, which may be made to coincide by a displacement after turning one of them over.

In the American texts the congruency of figures is discussed without reference to turning over.

As an example of the above discussion we present a study of "angles opposite the equal sides of an isosceles triangle."

It is given that ABC is an isosceles triangle with AB equal to AC. This triangle is turned over so that it assumes the position $A'B'C'$ in Figure 1. The pairs of angles B and B', C and C', and A and A' are equal. The triangle $A'B'C'$ is then glided in its plane. The angle $C'A'B'$ can be made to fall upon its equal BAC. Then because the sides are equal, $A'C'$ falls upon AB and $A'B'$ upon AC. The two triangles coincide, and the angles B and C' are equal. Therefore, the angles B and C are equal. The following statement can now be made:

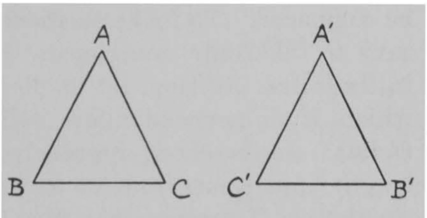

Fig. 1

Theorem.—In an isosceles triangle, the angles opposite the equal sides are equal.

In most of the American texts this theorem is proved by drawing the bisector of the vertical angle, thus forming two congruent triangles.

Another example of a proof by rotation is found in the theorem comparing the distances from the center of two equal chords of the same circle.

Let AB and $A'B'$ be two equal chords of the circle O (Fig. 2), and OC and OC' their respective distances from the center.

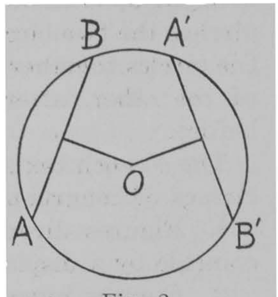

Fig. 2

It has been shown before that the center of the circle lies on the perpendicular bisectors of the chords. We now rotate the circle upon itself, in such manner that the arcs AB and $A'B'$ are placed in coincidence. Hence, the chords AB and $A'B'$ coincide, and also C and C', and OC is equal to OC'.

Let us now compare the chords of a circle when they are unequally distant from the center.

In order that the hypothesis may be apparent on the figure, it is expedient to arrange the chords perpendicular to the same radius. Then their distances from the center can be compared. To make the demonstration clear it is necessary to take only one chord. Keeping this chord parallel to its initial position, let us displace it along the radius to which it is perpendicular. Its mid-point describes this radius. As the chord approaches the center of the circle its length approaches that of a diameter, and hence becomes greater; as it recedes from the center its length approaches zero, for the chord approaches the position of a point at the outer end of the radius. Hence we may conclude:

> Theorem.—Of two unequal chords of the same circle or of equal circles, the greater is that which is nearer to the center.

Let us examine the French method of finding the sum of the angles of a triangle. It will be recalled that our texts solve this problem by drawing a line parallel to a side of the triangle through the opposite vertex, then comparing angles.

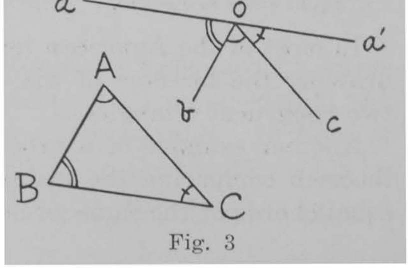

Fig. 3

Let ABC (Fig. 3) be a triangle. Let us begin by constructing around a point three adjacent angles respectively equal to the angles of the triangle. Two rays, *ob* and *oc*, are drawn through the point *o* parallel to *AB* and *AC* and of the same sense. The angle *boc* is equal to angle *A* of the triangle, for their sides are parallel and of the same sense. Now let us consider angle *B*. The ray *ob* is parallel to *BA* and of the opposite sense. If we draw *oa* parallel to *BC* and of the opposite sense, we form an angle *aob* equal to angle *B*.

Likewise, *oc* is parallel to *CA* and of the opposite sense. By drawing *oa'* parallel to *CB* and of the opposite sense, we form angle *a'oc* equal to angle *C*.

The two rays *oa* and *oa'*, being both parallel to *BC*, are in prolongation.

The result then is read upon the figure:

> Theorem.—The sum of the angles of a triangle is equal to two right angles.

We shall now present the notion of the *arc capable* of an angle, as given in some French texts.

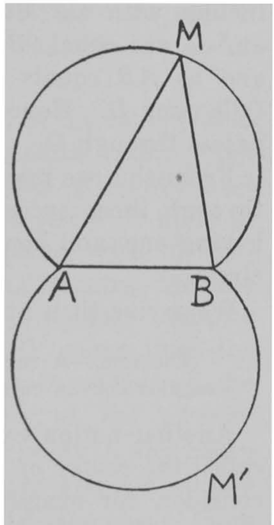

Fig. 4

If there is given triangle *AMB* (Fig. 4), and if a circle is circumscribed around this triangle, with only the arc *AMB* of this circle retained, then as the point *M* describes the arc the angle *M* remains constant in size.

If the figure is folded about *AB* as an axis, we find a second arc *AM'B* equal to the first one.

We shall say that from any point of these two arcs there is "seen" the segment *AB* under the angle *AMB*. This expression has its origin in the image of an eye placed at *M* which looks at the segment *AB*.

We shall then state:

> Theorem.—There being given two arcs of circles having a common chord *AB* and it being possible for them to coincide when the figure is folded along *AB*, from all the points of

these arcs, the segment AB is seen under a constant angle.

From any other point of the plane, the segment AB is not seen under this angle.

Definition.—The arcs above are called *arcs capables* of the angle AMB and of the extremities A and B.

The following method of showing that a circle may be circumscribed about a reguar polygon is not familiar to the American student.

Consider a regular convex polygonal line, $ABCDE$ (Fig. 5), and construct the circle O passing through the three vertices A, B, C. The diameter OI is drawn perpendicular to BC at its mid-point. Now let us fold the figure around OI as an axis. From truths already established it may be concluded that point B coincides with C. Since the angles B and C are equal, BA falls upon CD, and as AB equals CD, the point A falls upon D. Hence $OA = OD$, and the circle considered passes through D.

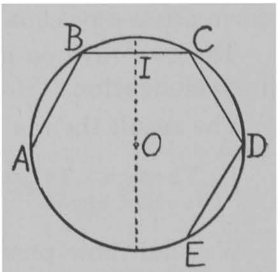

Fig. 5

From this we may conclude that the circle which passes through three successive vertices passes through the following one, and hence all the vertices are situated upon a circle.

We arrive then at the following:

Theorem.—A regular convex polygonal line and, hence, a regular convex polygon are inscriptible in a circle.

Another notion very unusual in our texts is that which is called the *center of repetition* of a regular polygon. Let us consider, for example, a regular pentagon inscribed in a circle. If we draw radii to the vertices of the polygon, there will clearly be five equal central angles. Then if the polygon is made to turn around the center through an angle equal to one of the central angles, each vertex will fall upon the following one, and the polygon will coincide with itself. If this operation is done five times, each vertex will come back to its initial position. This is expressed by saying that the center is for the polygon a center of repetition of order 5.

If the polygon should be a regular hexagon, there would be found a center of repetition of order 6.

The above notion is applied to determining the area of a regular polygon of n sides. For, since a regular polygon of n sides admits a center of repetition of order n, it is the sum of the assemblages of n equal isosceles triangles about this center. Thus a regular octagon is composed of eight equal triangles, each one having for an altitude the apothem of the polygon. Then the area of the polygon is 8 times the area of each triangle, which reduces to 8 times the product of a side and one-half the apothem.

In the French texts under examination, the notion of rectilinear translation seems to be a very useful device for demonstrating propositions of almost all types. As an example, it can be shown by this method that two circles of the same radius are congruent.

Suppose we have two circles, O and O', in the same plane, with equal radii, r and r' (Fig. 6). We first make circle O' glide in its plane, with its center O' tracing the line OO', in such manner that O' comes upon the point O. Then all the points of the circumference O' are at a distance $r' = r$ from point O, and hence lie upon the circumference O. In a like manner all the points of the circumference O are at a distance $r = r'$ from the point O'. Therefore, the two circumferences coincide and are congruent.[5]

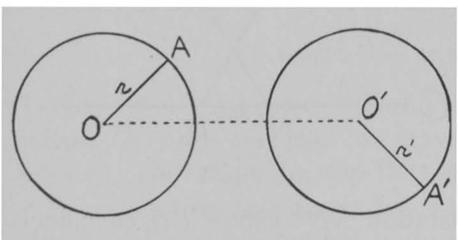

Fig. 6

Under the discussion of rectilinear translations we find the following definition of parallel lines:

[5]Chenard, H.: *Géométrie*, p. 37.

Two lines being given, if, by the aid of a rectilinear translation of either one of them, they can be made to coincide, it is said that these two lines are parallel.

Under the general properties of parallel figures we find the following:

Theorem.—If the figures (F) and (G), which may be two lines, two planes, or a plane and a line, are parallel by the aid of a translation MN, M being a point of (F) and N a point of (G), they are again parallel by the aid of a translation $M'N'$, M' being any point of (F) and N' any point of (G).

We take the case of two lines:

Let (F) be represented by AB and (G) by CD, and let AB and CD be parallel by the aid of the translation MN (Fig. 7).

To show that AB and CD are likewise parallel by the translation $M'N'$, M' being any point of AB and N' any point of CD.

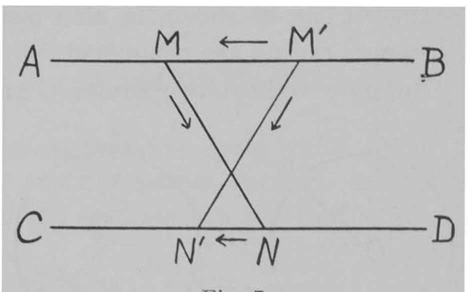

Fig. 7

Let us impose upon AB a first translation $M'M$, after which AB remains in coincidence with itself; a second translation MN makes AB come upon CD, by hypothesis; a third translation NN' leaves CD in coincidence with itself. The three successive translations, $M'M$, MN, NN', result in placing AB upon CD. Then likewise their resultant $M'N'$ makes AB and CD coincide, and the two lines AB and CD are parallel by the aid of the translation $M'N'$, by the following postulate, which has been given to the student before:

If any n successive translations, MM_1, M_1M_2 . . . $M_{n-1}M_n$, make a figure to pass from an initial position F to a final position F_n, the single translation MM_n makes the same figure pass directly from the initial position F to the final position F_n.

We wish to present here a theorem illustrating the way in which the French make use of the notion of symmetry in developing geometric proofs. This theorem is, of course, given only after certain fundamental truths regarding symmetry have been established in the minds of the pupils.

Theorem.—If two lines are symmetrical with respect to a point, they are parallel.

Let AB and $B'A'$ be two lines symmetrical with respect to the point O (Fig. 8).

To prove that AB and $B'A'$ are parallel.

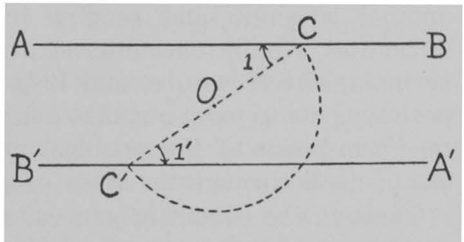

Fig. 8

If we make the plane ABO turn through 180° around the point O, AB comes upon $A'B'$. Any point C of AB then comes upon C' of $A'B'$, so that OC' and OC are in a straight line, and during the rotation of ABO angle 1 remains constant and comes upon angle 1′, so that

$$\text{angle } 1 = \text{angle } 1'$$

Then if we consider AB and $A'B'$ as lines cut by the secant CC', we see that we have equal alternate-interior angles. Therefore the two lines are parallel.

In concluding this paper we wish to say that space and time have been too limited to do more than give a cursory presentation of the methods of teaching geometry in France, leaving the reader, in most instances, to make his own inferences and comparisons. Also, there has been no attempt to disparage the teaching of mathematics in our own country. In an investigation which has not been too thorough, nor conducted upon scientific principles, it is easy to become over-enthusiastic and to mistake the ideal for the real. But to one uninitiated into this particular field of exploration, this limited experience has been a happy one.

AN EXPOSITORY NOTE ON THE e-SYSTEMS

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Introduction. On account of the widespread interest in relativity and the prominent position of this theory in modern physics, it is quite likely that a fair proportion of the students of mathematics and physics will, at one time or another, attempt some reading in this field. Among the difficulties usually encountered are some of a mechanical or manipulative nature, and it is the purpose of this expository note to point out that a first step toward mastering the formal side of the calculus involved (tensor analysis) can be made through the study of determinants.

Through the advent of general relativity in 1916 the investigation of tensors, particularly as a means of constructing generalized differential geometries, received a great impetus. Applications have been made to a variety of subjects, a recent instance being the treatment of certain problems connected with electrical machinery. At present, it is the opinion of many of those in a position to judge, that this calculus is destined to occupy a rather dominant position in the field of mathematical physics. The success of this instrument is due, for the most part, to its power to furnish invariant descriptions of situations in geometry and physics. Among its other salient characteristics are operational simplicity and compactness. Perhaps some of the material to be introduced will suffice as an illustration of this last quality.

Notation and operations. Essentially, an e-system is a set of symbols for the numbers 0, +1, —1. For example, the e-system of order two (e^{rs}) consists of the 2^2 particular symbols defined by the equations:

$$e^{11} = 0; \; e^{12} = 1; \; e^{21} = -1; \; e^{22} = 0;$$

or, more briefly, by the statements: e^{12} is unity; e^{rs} is skew symmetric (i.e., $e^{rs} = - e^{sr}$). The superscripts on e and in the symbols to be introduced are used in the same manner as subscripts, that is, as distinguishing marks. Thus in the determinant A,

$$A = \begin{vmatrix} a^1{}_1 & a^1{}_2 \\ a^2{}_1 & a^2{}_2 \end{vmatrix},$$

we use the superscripts and subscripts to indicate the row and column numbers, respectively. Let us note in passing that, by virtue of the foregoing definitions, the sum $\sum_{rs} e^{rs}a^1{}_r a^2{}_s$, which is $e^{11}a^1{}_1a^2{}_1 + e^{12}a^1{}_1a^2{}_2 + e^{21}a^1{}_2a^2{}_1 + e^{22}a^1{}_2a^2{}_2$, reduces to $a^1{}_1a^2{}_2 - a^1{}_2a^2{}_1$, the expansion of the determinant.

A second matter of notation is the abridgment of omitting the summation sign from expressions such as $\sum e^{rs}a^1{}_r a^2{}_s$. More precisely, if a term contains the same lower-case letter (excluding n) both as a superscript and as a subscript, then we shall understand that this term represents the sum of all of the particular terms obtainable by assigning to each repeated index a value from the prescribed set 1, 2, . . . , n. To illustrate, if $a^r{}_s$ and $b^r{}_s$ are elements of nth order determinants, then $a^r{}_s b^s{}_t$ is $a^r{}_1 b^1{}_t + a^r{}_2 b^2{}_t + \ldots + a^r{}_n b^n{}_t$, the product of the rth row of the first and the tth column of the second. Likewise, if, in addition, $b_1, b_2, \ldots, b_n$ is a set of given numbers and $x_1, x_2, \ldots, x_n$ a set of unknowns, then $a^r{}_s x_r = b_s$ is a set of n linear equations in n unknowns.

Now by virtue of the laws of arithmetic there is a variety of schemes for expanding summations, the sole requirement being that we are to form the sum of all of the particular terms obtainable by assigning to the repeated indices values from the prescribed set $1, 2, \ldots, n$. For example, parentheses may be inserted in a term at will, thus $a^{rs}b_r c_s = (a^{rs}b_r)c_s = (a^{11}b_1 + a^{21}b_2 + \ldots + a^{n1}b_n)c_1 + (a^{12}b_1 + a^{22}b_2 + \ldots + a^{n2}b_n)c_2 + \ldots + (a^{1n}b_1 + a^{2n}b_2 + \ldots + a^{nn}b_n)c_n$ which, after applying the distributive law, is a particular expansion of $a^{rs}b_r c_s$. Frequently, we can verify identities by merely conceiving the two members to be written out according to the same scheme and noting the equality of the corresponding terms, the term to be expanded being regarded for the moment as a typical particular term of its expansion. As an illustration, consider the identity: $e^{rs}(b^1{}_r + c^1{}_r)a^2{}_s = e^{rs}b^1{}_r a^2{}_s + e^{rs}c^1{}_r a^2{}_s$.

The third order e-systems are defined as follows: $e^{rst} = e_{rst}$; $e^{123} = 1$; e^{rst} is skew symmetric in each pair of indices. Thus e^{rst} is zero if any two indices are the same and otherwise is one or minus one according as r,s,t is an even or an odd permutation of 1,2,3 (if r,s,t can be transformed into 1,2,3 by an even number of interchanges of the elements of pairs, then every such transformation will contain an even number of interchanges and the permutation r,s,t will be said to be even).

Closely allied with the e-systems are the Kronecker deltas: $\delta^i{}_r{}^j{}_s{}^k{}_t = e_{rst}e^{ijk}$; $\delta^i{}_r{}^j{}_s = \delta^i{}_r{}^j{}_s{}^k{}_k = \delta^i{}_r{}^j{}_s{}^1{}_1 + \delta^i{}_r{}^j{}_s{}^2{}_2 + \delta^i{}_r{}^j{}_s{}^3{}_3$; $\delta^i{}_r = \frac{1}{2}\delta^i{}_r{}^j{}_j$. With regard to the second delta, we observe that if i and j are distinct then one and only one of e^{ij1}, e^{ij2}, e^{ij3} differs from zero, and consequently if i,j and r,s are not the same set of numbers then $\delta^i{}_r{}^j{}_s$ vanishes. Furthermore, if $i = r$, $j = s$, $i \neq j$, then $\delta^i{}_r{}^j{}_s = 1$; whereas if $i = s$, $j = r$, $i \neq j$, we have $\delta^i{}_r{}^j{}_s = -1$. To recapitulate, $\delta^i{}_r{}^j{}_s$ vanishes if $i = j$, or $r = s$, or if the set i,j is not the set r,s and is otherwise plus or minus one according as r,s is an even or an odd permutation of i,j. Obviously, $\delta^i{}_r$ vanishes if i is not r, while it is unity if i and r are the same number. According to the summation convention, however, the symbol $\delta^r{}_r$ represents $\delta^1{}_1 + \delta^2{}_2 + \delta^3{}_3$, which is three.

Determinants. It will be recalled that the determinant A,

$$A = \begin{vmatrix} a^1{}_1 & a^1{}_2 & a^1{}_3 \\ a^2{}_1 & a^2{}_2 & a^2{}_3 \\ a^3{}_1 & a^3{}_2 & a^3{}_3 \end{vmatrix},$$

is defined to be the sum of all of the products $\pm\, a^1{}_r a^2{}_s a^3{}_t$ obtainable by selecting one and only one element from each column and prefixing a plus or minus sign according as r,s,t is an even or an odd permutation of 1,2,3. Thus, in the notation we have adopted,

$$A = e^{rst}a^1{}_r a^2{}_s a^3{}_t \quad .$$

THEOREM: $e^{rst}a^i{}_r a^j{}_s a^k{}_t = Ae^{ijk}$.

Proof: It may be observed that if we replace i,j,k with 1,2,3 in the statement of the theorem the result, by virtue of certain of the preceding definitions, is A. Consequently, it will be sufficient to prove that the left member is skew symmetric in each pair of the free indices i,j,k. Now by successively interchanging the dummy indices s,t; permuting the symbols $a^j_t a^k_s$; and replacing e^{rts} with $—e^{rst}$; we may write

$$e^{rst}a^i_r a^j_s a^k_t = e^{rts}a^i_r a^k_s a^j_t = —e^{rst}a^i_r a^k_s a^j_t \quad .$$

The skew symmetry of the other pairs may be established in an analogous manner. Incidentally, it will aid us in developing expressions for the cofactors if we note that quantities such as $e^{rst}a^j_s a^k_t$ are likewise skew symmetric in their free indices.

To convince ourselves of the truth of the similar theorem $e_{ijk}a^i_r a^j_s a^k_t = Ae_{rst}$ it will suffice perhaps to verify that $e_{ijk}a^i_1 a^j_2 a^k_3$ is A. Accordingly, let us note that if we permute the factors in $a^i_1 a^j_2 a^k_3$ until we obtain $a^1_r a^2_s a^3_t$, the permutation r,s,t will be even or odd according as i,j,k is even or odd, and so $e^{rst} = e_{ijk}$ and therefore

$$e_{ijk}a^i_1 a^j_2 a^k_3 = e^{rst}a^1_r a^2_s a^3_t = A \quad .$$

THEOREM (on interchanging rows and columns): If $a^r_s = b^s_r$, then $e^{rst}a^1_r a^2_s a^3_t = e^{rst}b^1_r b^2_s b^3_t$.

Proof: $e^{rst}a^1_r a^2_s a^3_t = e_{rst}b^r_1 b^s_2 b^t_3$.

THEOREM (on interchanging rows): If $a^1_r = b^2_r$; $a^2_s = b^1_s$, then $e^{rst}a^1_r a^2_s a^3_t = — e^{rst}b^1_r b^2_s b^3_t$.

Proof: $—e^{rst}b^1_r b^2_s b^3_t = —e^{rst}a^2_r a^1_s a^3_t = e^{srt}a^1_s a^2_r a^3_t$.

THEOREM (on the addition of certain determinants): $e^{rst}(b^1_r + c^1_r)a^2_s a^3_t = e^{rst}b^1_r a^2_s a^3_t + e^{rst}c^1_r a^2_s a^3_t$. (Obvious).

THEOREM (on the multiplication of determinants): If $A = e^{rst}a^1_r a^2_s a^3_t$, $B = e^{ijk}b^1_i b^2_j b^3_k$, then $AB = e^{ijk}(a^1_r b^r_i)(a^2_s b^s_j)(a^3_t b^t_k)$.

Proof: $(e^{ijk}b^r_i b^s_j b^t_k = B\ e^{rst})a^1_r a^2_s a^3_t$.

THEOREM: (on the differentiation of determinants): If each a^i_j has a finite derivative in a region R, then in R, $De^{rst}a^1_r a^2_s a^3_t = e^{rst}(Da^r_1)a^2_s a^3_t + e^{rst}a^1_r(Da^2_s)a^3_t + e^{rst}a^1_r a^2_s (Da^3_t)$.

Proof: This theorem is an obvious identity by virtue of the rules for differentiating sums and products.

Before considering the next theorem let us endeavor to express the cofactors of A in the notation which we have adopted. Now the cofactor (A^s_r) of the element a^r_s of a determinant A is the coefficient of a^r_s in the expansion $e^{rst}a^1_r a^2_s a^3_t$; thus

$$A^r_1 = e^{rst}a^2_s a^3_t = \tfrac{1}{2}(e_{123}e^{rst}a^2_s a^3_t + e_{132}e^{rst}a^3_s a^2_t) = \tfrac{1}{2}e_{1jk}e^{rst}a^j_s a^k_t = \tfrac{1}{2}\delta^{r\,s\,t}_{1\,j\,k}a^j_s a^k_t$$

$$A^s_2 = e^{rst}a^1_r a^3_t = \tfrac{1}{2}(e_{123}e^{rst}a^1_r a^3_t + e_{321}e^{rst}a^3_r a^1_t) = \tfrac{1}{2}\delta^{r\,s\,t}_{i\,2\,k}a^i_r a^k_t = \tfrac{1}{2}\delta^{s\,r\,t}_{2\,i\,k}a^i_r a^k_t \ .$$

Thus, in general, $A^r_i = \tfrac{1}{2}\delta^{r\,s\,t}_{i\,j\,k}a^j_s a^k_t$.

THEOREM: $a^i_u A^r_i = A\delta^r_u$; $a^u_r A^r_i = A\delta^u_i$.

Proof: $a^i_u A^r_i = \tfrac{1}{2}(a^i_u e_{ijk}a^j_s a^k_t e^{rst} = \tfrac{1}{2}Ae_{ust}e^{rst} = A\delta^r_u$,etc.

THEOREM: If A is not zero and $a^u_r x_u = b_r$ has a solution, then it has a unique solution.

Proof: $A^r_i b_r = (A^r_i a^u_r)x_u = A\delta^u_i x_u = Ax_i$.

It should be noted that "multiplication" of $a^u_r x_u = b_r$ by A^r_i, on account of the repeated index, means the sum of the results obtained by multiplying the first equation by A^1_i, the second by A^2_i, and the third by A^3_i ; also $A^r_i b_r/A$ is, of course, single valued.

THEOREM: If A is not zero, then the set of numbers $A^r_i b_r/A$ is the solution of $a^i_s x_i = b_s$.
Proof: $a^i_s A^r_i b_r/A = \delta^r_s b_r = b_s$.

If A is not zero, then the quantities A^r_i/A are called the normalized cofactors and are designated by α^r_i. Evidently, $\alpha^r_i a^i_s = \delta^r_s$, $\alpha^r_i a^j_r = \delta^j_i$.

THEOREM. If A is not zero and $B^r{}_i$ is a set of 3^2 numbers, such that, for each r, $B^r{}_i a^s{}_r = \delta^s{}_i$, then $B^j{}_i = \propto^j{}_i$.

Proof: $B^j{}_i = B^r{}_i \delta^j{}_r = B^r{}_i a^s{}_r \propto^j{}_s = \propto^j{}_s \delta^s{}_i = \propto^j{}_i$.

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MODERN TENDENCIES IN THE TEACHING OF MATHEMATICS*

BY E. E. HEIMANN

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If we agree that it is the function of an education to prepare an individual to function most effectively in a changing social order, and to adapt him to a progressive civilization, we must also agree that, as the social order changes and becomes more and more complex, the necessary educational equipment to meet these changes must change. Consequently our school curricula must change and be modified from time to time to meet the demands that a changed environment makes on our educational system. This being true, it is necessary for every subject in the curriculum to adapt itself to such changes and to offer what the new order demands, or else to give up its place in the curriculum to some other subject that meets those demands.

The science of mathematics offers certain peculiarities of its own that have prevented as much change in it as there has been in many other subjects. In the first place, mathematics is the most exact science. Many of its fundamental concepts and truths have been known for many centuries. A truth, in order to be a truth, is true under all conditions and for all time. The mathematical facts discovered by our forefathers are still as true as they ever were. The geometry of Euclid, for instance, as taught today, is scarcely more rigorous than it was 2,000 years ago. It is true, however, that the applications of mathematics change with the changes in our physical environment, and that the amount of mathematical training required changes with the demands of the times and with the changes in the personnel of the student body that is given this mathematical training. Since our problems in college mathematics are so closely related to the work that is done in high school, a few

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*Read before the Oklahoma Section of the Mathematical Association of America at Tulsa, February 1, 1935.

words about the changes that have taken place in the high school mathematical instruction will not be amiss.

In the first place, the high school student of today differs in age, interest, and experience from the high school student twenty to twenty-five years ago. The high school then was almost as exclusive as the college of today; that is, only a relatively small fraction of all pupils went to high school then, while today practically all children go to high school. The modern attitude toward high schools is that they should take all pupils that want to enter, and then keep them there until they graduate. Consequently, high schools are forced to accept pupils who can learn only the most elementary things, and they have to graduate pupils who are altogether unable to do college work. With this large and mixed group on its hands, it is the problem of the high school of today to provide something in its curriculum for each individual member of this group. To do this, high schools have broadened their curricula and are now offering a wide variety of elective courses so that the high school graduate, when he enters college, represents no particular standard of attainment either in quantity or quality of work done. These changes have had their effect on high school mathematics as well as on most other subjects. The requirements for graduation have been lowered. In addition to that, educators have stressed the importance of making the high school course more interesting and practical by making it deal with practical problems of present value to the student. As a result, the important mechanical processes that lay the foundation for college mathematics are no longer taught as thoroughly as they formerly were, and many are omitted altogether.

As a result of these changes in the high school, there is an almost universal complaint on the part of colleges that high school pupils come to college poorly prepared to do college work in mathematics. The per cent of failures in freshman courses of college mathematics is high; higher, in fact, than in any other subject. This statement is not based merely on personal opinion, but I have statistics to bear this out. I am sorry that I do not have these statistics for Oklahoma colleges, but I have them for the adjoining state of Texas, where conditions ought to be at least somewhat similar to ours. I have these statistics from a report of the Southern Association of Secondary Schools and Colleges, and they are contained in an article entitled "Does the Present High School Curriculum in Mathematics Prepare Students for College?"; an article written by Charles R. Sherer of Texas Christian University and published in the Texas Mathematics Teachers Bulletin of February 8, 1934. The report referred to gives the average per cent of failures in freshman mathematics for all colleges of Texas for a four-year period. To show that the per cent of failures in mathematics is much higher than in any other subject, that for a few other subjects was also given. The report showed that the per cent of failures in history was 13.6; in French, 14.3; in English, 14.4; in science, 19; and in mathematics, 26.2. You see that there were nearly twice as many failures in mathematics as in history, and that the number of failures in science ranked next to mathematics, a result to be expected in view of the mathematical nature of the sciences. It seems, then, that the present high school curriculum does not adequately prepare students for college.

What, then, are some of the things that we can do and what are some of the things that are being done? As high schools now require fewer units in mathematics for graduation, there has been a tendency in colleges to lower their entrance requirements in mathematics or to do away with them altogether. There is also a tendency in colleges to no longer require any mathematics for a bachelor's degree except in certain fields of work. At the present time, however, a large number of colleges still require that all students take from six to eight hours. We are confronted, then, with the problem of where to begin in college with a group that vary so much in the mathematical foundation that they have. There will be a small group who will have had four years of high school mathematics, including trigonometry and solid geometry, but there will be a large group who will have had no real foundation for college mathematics at all. And yet, there should be a definite starting point for college mathematics. With the lowered requirements for high school graduation, there is danger of colleges lowering their standards in order that poorly prepared high school graduates may be able to do freshman work in college. Also, if we require the same number of hours of college mathematics of all students, we are apt to punish those students who have had the most high school mathematics, for they will have to start in college with work much more advanced than that which those students will do who have had very little high school mathematics. To make it possible, therefore, to begin all college freshmen at the same level, and at the same time to make it possible to take high school students with little preparation, many colleges are now offering courses which are generally called "Zero courses," for which no credit is given, and whose function is to make up the deficiencies that freshmen may have. In view of the fact that high schools stress particularly the present value of the subject, thereby failing to ground the student as thoroughly in the mathematical processes as is desirable for college work, such zero courses have the additional value of making the student proficient in these operations.

It might be of interest in this connection to mention an article by Dr. Hart of the University of Minnesota which appeared in the American Mathematical Monthly for December. It is entitled "Student Placement in Secondary Mathematics," and in it he expresses the opinion that two and a half units of high school mathematics should be the very minimum for college entrance. He bases this opinion on the fact that so many branches of knowledge besides the physical sciences now use the mathematical, especially the statistical, approach to the subject, and that unless a student has the necessary foundation in mathematics, he is deprived of many courses that he could otherwise take.

He further suggests that high schools should make an attempt to separate students into two groups at an early stage in their high school career—those who are likely to enter college or who are capable of doing college work in one group, and those who are not likely to ever enter college in the other group. For the first group he recommends a rather rigid course of prescribed subjects that adequately prepare for college entrance, whereas the second group could pursue a vocational course. If this plan could be put into operation, the problem that I raised above would be solved.

There are various other ways in which some colleges meet this problem. To maintain high standards for their majors in mathematics, many schools make extra requirements of them. In some, certain courses may be counted toward a degree by students not majoring in mathematics but not by majors in that subject. Others name certain advanced courses that all major students must take. Still others specify the number of hours of senior college work that a student must take. George Washington University, in Washington, D.C., requires that a student majoring in mathematics take fifteen semester hours of mathematics above the first course in calculus.

The question of whether or not mathematics should be required of all college students is a debatable one, and I shall not attempt to answer it here. Along with it is often raised the question of a general course. If mathematics is required of all students, the majority will not be interested in any one particular course, and a general course would probably be of most value to them. It is my opinion that a worth while general course can be given after the student has studied some of the elementary branches of mathematics. The University of Chicago offers such an introductory general course in the physical sciences. However, of a total of ninety lectures, only ten are devoted to mathematics, whereas twenty-two are devoted to physics and twenty-three to geography and geology. When one looks at the examination that they give at the end of this course, however, one finds that it requires a knowledge of algebra through theory of equations, trigonometry, analytic geometry, and differential calculus, so that it covers almost the entire field of junior college mathematics. A more practical course for those not interested in the field of mathematics or in the physical sciences is offered by the Colorado State Teachers College. This course is called "Informational Mathematics" and is required of all students not majoring in mathematics in their sophomore year. It is a four-hour course and includes topics such as the following: number concepts and number systems; early notation; finding an average, a median, a mode; how we measure; degree of accuracy in measurements and in computations from measurements; mathematics of the home, budgets, accounts, etc.; business mathematics, bonds, investments, legal papers, insurance, banking, etc.; the nature of algebra, the formula, the equation, the graph; the nature of geometry, Euclid and Pythagoras; Newton and Leibnitz and the calculus; and many other topics. You see that it is a very general course, but that it teaches the mathematics that the average person needs most.

Another tendency in the college mathematics curriculum is to offer more courses dealing with business mathematics. Looking through the catalogs of various colleges one finds many such courses listed. There are, for instance, courses in statistics, the mathematics of investment, mathematics of life insurance, applied mathematical groundwork of economics, mathematics of finance, etc. The importance of business mathematics in the average person's life is recognized, and it is right that we should give it a place in our college curriculum. As I mentioned before, the statistical approach to many subjects is now used, and it is desirable that the groundwork for such work be given in separate courses in statistics.

Finally, with the change in the curriculum and in the student body, there has been also some change in the mathematics faculties of colleges, and in the methods of teaching mathematics. Colleges are beginning to recognize the importance of teaching ability, or teaching skill, in addition to knowledge of subject matter. A person may be a great mathematician or a great scientist and yet be a very poor teacher. The very fact that students come to college at a much earlier age than they used to, and the fact that they are poorly prepared in most instances, makes it necessary that we give our students more personal attention and guidance. They simply have not learned to shift for themselves, and must be gradually taught to do so. Our method of instruction, therefore, has undergone changes. The lecture method, if it was ever justifiable in mathematics, is not so any longer, at least not in junior college courses. It is often said that one learns by doing, and nowhere is that more true than in the study of mathematics.

In conclusion, let me briefly summarize the points I have attempted to bring out. The greatest problem facing colleges today is that of working out a closer correlation between high school and college mathematics so that a student may be able to continue his mathematical training in college without any loss of time. We must maintain our college standards by stating definitely where college mathematics shall begin, and by refusing to give credit for work below that level. I have recommended a general course for those students not particularly interested in any one branch of college mathematics. It has been observed that there is a tendency for colleges to offer more courses dealing with business mathematics. And finally, with changes in the student personnel, we found that methods of teaching have changed and that new emphasis is placed on teaching skill.

NOTE: In the discussion following the reading of the paper, Dr. Nathan A. Court of the University of Oklahoma made an interesting statement with reference to the large number of failures in mathematics as compared with other subjects. He pointed out that there are two main reasons for this. First, mathematics is cumulative; that is, everything that a student learns at any stage of his education he will have to remember and be able to use later on. Therefore what he has failed to get at any one stage will always handicap him. This is not true of other subjects, or at least not to such an extent. Second, mathematics is more exact and we can rate a student more exactly than in any other subject. He either knows it or he doesn't, and he cannot hide the fact from his instructor. For these reasons, he thinks, and not because we in the mathematics field are poorer teachers, do we have more failures in mathematics.

WHAT I WOULD LIKE MY PHYSICS STUDENTS TO KNOW WHEN THEY COME TO MY CLASS

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More important than what they *know* is what they *want* to know. I am a strong believer in the statement, "whatever a persons wants he will get if his wants are strong enough and his backbone more active than his wishbone." Nevertheless, a good foundation is an asset, so we'll get back to our topics. Three things are invaluable to a student of physics: he should read intelligently; he should think logically; and he should use numbers accurately. Since the first two apply to any course, we will limit this paper to number sense.

Physical laws are dependent in large measure upon mathematics for their applications. Since many are reduced to strictly mathematical formulas, their experimental verification will depend entirely upon the use of numbers. A thorough knowledge of number combinations and the four fundamental operations is presupposed in any physics course. It should not be necessary for a high school student of physics to waste mental energy during class and laboratory periods on these things; they should be familiar tools ready at a moment's notice.

Frequently in using formulas it is necessary for the student to know how to factor, how to substitute unknown values, and how to analyze problems. The ability to simplify fractional expressions and to solve equations for unknowns is essential. Decimals and percentage should have been mastered by the seventh grade child before he enters high school. The knowledge of quadratics is frequently needed. Ratio and proportion and denominate numbers should not be strangers to the high school senior. He will find it to his advantage to have a working knowledge of triangle and circle theorems for his physics problems.

Sometimes students get into the physics classes without this knowledge, and then the instructor must turn mathematics teacher at the expense of his own subject. It is often necessary for him to take time out while demonstrating the laws of the pendulum to teach a student to find the square root of a number. Many times must he stop in the midst of calorimetry experiments to teach (not merely review) the principle of parenthesis removal and the method of solving some equation. Another annual experience is having to stop an interesting experiment on image formation and revert to ninth grade algebra fractions and plane geometry triangle theorems. A very annoying experience of frequent recurrence is the interruption of an explanation of Archimedes' Principle to explain for eleventh grade students the method of calculating the volumes of regular cylindrical or rectangular objects.

But who are we of the Science Department to sit back and criticize the work of the Mathematics Department? We have our own shortcomings. We should perhaps accept things as they are and precede each unit in physics with a lesson or lessons on factoring, fractions, square roots, substitutions, triangles, circles, or volume formulas as each is needed. Having done this, we could blame no one but ourselves when the necessary mathematical information is missing.

THE BROWN UNIVERSITY PRIZE EXAMINATION

The Brown University Prize Examination for freshmen was given on Saturday, October 13, 1934. The three prizes were awarded as follows:

First: Roy Howard Baskin, of Cameron.
Second: Paul Eugene Cooper, of Galveston.
Third: Julian Milton Meer, of San Antonio.

Good papers were also written by Giles C. Avriett of Cameron, Gordon H. Fisher of San Benito, and L. De Witt Hale of Farmersville.

The questions were as follows:

1. One side of a rectangular room is 2 feet longer than the side of a square room in the same house. The perimeter of the first room is 30 feet less than twice the perimeter of the second, and the floor-space of the first is 8 square feet more than twice that of the second. What would it cost to carpet both rooms if the carpet costs $2.00 per square yard?

2. An army train 10 miles long starts forward at a constant rate at the same time that a cyclist starts forward, at a constant rate, from the foot of the train. The cyclist catches up with the head of the train, then immediately turns around, travels toward the foot, meets the foot of the train at the point from which the head of the train started. How far did the cyclist travel in all?

3. Construct a circle tangent to a given circle at a given point and also tangent to a given line.

4. From the centers of each of two non-intersecting circles tangents are drawn to the other circle. Prove that the chords A_1B_1 and A_2B_2 are equal.